Polygon Laplacian Made Simple

Supplementary Material

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In this document we provide additional quantitative evaluations and comparisons for the applications described in the main paper. We report root-mean-square errors (RMSE) for mean curvature deviation (Table 1), reproduction of the spherical harmonics (Table 2), and geodesic distances (Table 3), the former two computed on spherical meshes (Figure 1) and the latter one on planar meshes (Figure 2).

We evaluate our polygon Laplacian, constructed with different choices of the per-polygon virtual vertex and with/without lumping of the mass matrix. We compare our results to Alexa and Wardetzky's operator [AW11] with different choices for their hyper-parameter λ. Additionally, we triangulate the polygons such that the sum of squared triangle areas is minimized, using the dynamic programming approach of Liepa [Lie03]. For geodesic distances, we also provide results for computing the Laplacian based on the intrinsic Delaunay triangulation [BS07] of this minimum area triangulation.

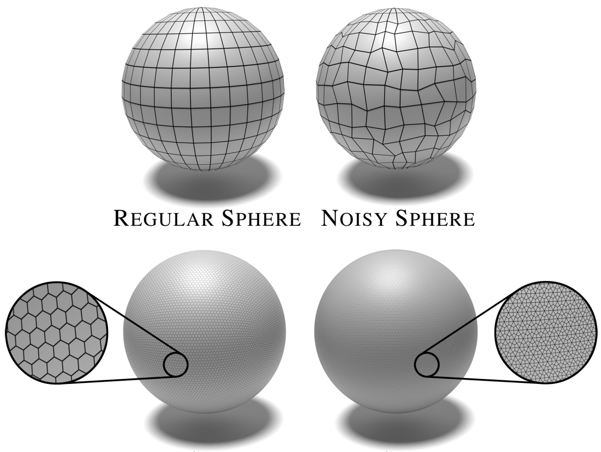

Figure 1: *Spherical meshes used for testing the accuracy of spherical harmonics and mean curvature.*

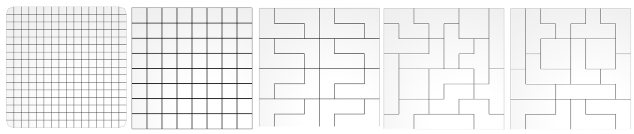

Figure 2: *Planar meshes used for the evaluation of geodesic distances, including non-convex and non-star shaped tessellations.*

References

[AW11] ALEXA M., WARDETZKY M.: Discrete Laplacians on general polygonal meshes. *ACM Transactions on Graphics 30*, 4 (2011), 102:1–102:10. 1, 2

[BS07] BOBENKO A. I., SPRINGBORN B. A.: A discrete Laplace–Beltrami operator for simplicial surfaces. *Discrete & Computational Geometry 38*, 4 (2007), 740–756. 1, 2

[Lie03] LIEPA P.: Filling holes in meshes. In *Proceedings of Eurographics/ACM SIGGRAPH Symposium on Geometry Processing* (2003), pp. 200–205. 1, 2

	un-lumped mass matrix				*lumped mass matrix*				[AW11]			
Mesh	Affine	Convex	Centroid	Abs.Area	Affine	Convex	Centroid	Abs.Area	$\lambda = 0.1$	$\lambda = 0.5$	$\lambda = 1$	[Lie03]
HEX SPHERE	0.0039	0.0159	0.0159	0.0100	**0.0016**	0.0049	0.0049	0.0026	0.0023	0.0038	0.0075	0.3711
FINE SPHERE	0.3669	0.4181	0.4181	0.5454	0.1334	0.1332	0.1332	0.1630	0.0279	**0.0107**	0.0141	0.0623
REGULAR SPHERE	0.0515	0.0515	0.0515	0.0537	**0.0168**	**0.0168**	**0.0168**	0.0172	0.0290	0.0290	0.0290	0.0469
NOISY SPHERE	0.01520	0.1463	0.1463	0.1641	0.0470	**0.0440**	**0.0440**	0.0493	0.0562	0.0814	0.1551	0.1053

Table 1: *RMSE of mean curvature computation on the spherical meshes in Figure 1.*

	un-lumped mass matrix				*lumped mass matrix*				[AW11]			
Mesh	Affine	Convex	Centroid	Abs.Area	Affine	Convex	Centroid	Abs.Area	$\lambda = 0.1$	$\lambda = 0.5$	$\lambda = 1$	[Lie03]
HEX SPHERE	9.18e-7	4.56e-6	4.56e-6	2.10e-5	**7.41e-7**	1.15e-6	1.15e-6	4.30e-6	9.93e-6	1.46e-6	1.13e-6	0.0037
FINE SPHERE	0.0009	0.0009	0.0009	0.0011	0.0003	0.0003	0.0003	0.0004	0.0005	**6.48e-5**	8.98e-5	0.0005
REGULAR SPHERE	0.0256	0.0258	0.0258	0.0272	0.0393	0.0398	0.0398	0.0398	0.0599	0.0220	**0.0175**	0.0200
NOISY SPHERE	0.0636	0.0804	0.0804	0.0623	0.0643	0.0644	0.0644	0.0655	0.0969	**0.0589**	0.1366	0.0722

Table 2: *RSME of spherical harmonics on the meshes in Figure 1.*

	un-lumped mass matrix				*lumped mass matrix*				[AW11]				
Mesh	Affine	Convex	Centroid	Abs.Area	Affine	Convex	Centroid	Abs.Area	$\lambda = 0.1$	$\lambda = 0.5$	$\lambda = 1$	[Lie03]	[BS07]
QUADS 1	0.025	0.025	0.025	0.025	0.026	0.027	0.027	0.027	**0.018**	0.042	0.120	0.039	0.039
QUADS 2	**0.031**	**0.031**	**0.031**	**0.031**	0.036	0.036	0.036	0.036	0.040	0.047	0.123	0.086	0.086
L-SHAPED	0.141	0.165	0.134	0.134	**0.057**	0.063	0.068	0.068	0.422	0.112	1.653	0.071	0.074
TETRIS 1	0.465	0.490	0.490	0.491	0.218	0.181	0.186	0.185	0.452	0.230	0.249	0.168	**0.141**
TETRIS 2	0.406	0.346	0.151	0.153	0.082	0.089	0.089	0.088	0.438	0.118	0.185	0.081	**0.067**

Table 3: *RMSE of geodesic distances for the planar meshes in Figure 2.*