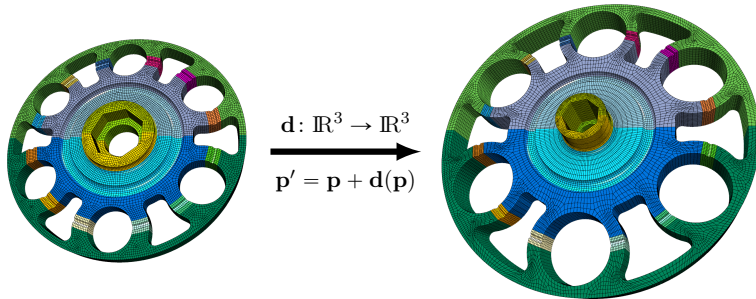


# High Quality Mesh Morphing Using Triharmonic Radial Basis Functions

Daniel Sieger, Stefan Menzel, and Mario Botsch



# Outline

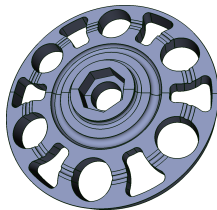
1. Introduction
2. RBF Volume Morphing
3. Surface Morphing
4. Results
5. Conclusion

# Introduction

- Mesh morphing for simulation-based design optimization
- Update simulation mesh according to updated CAD model

# Design Optimization Scenario

Initial Design

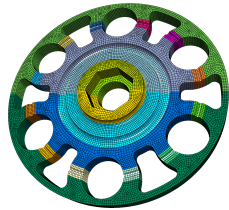


# Design Optimization Scenario

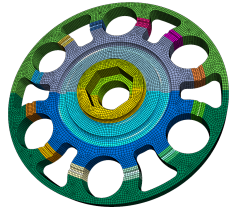
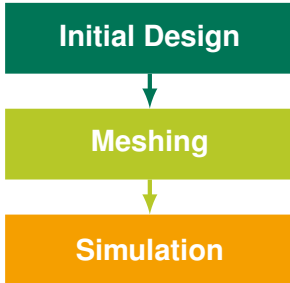
Initial Design



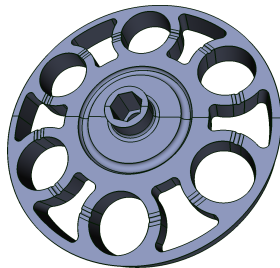
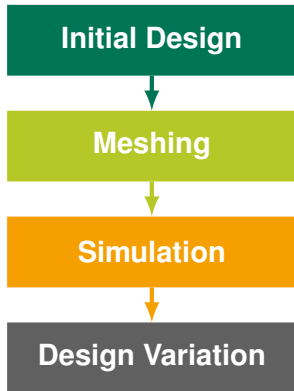
Meshing



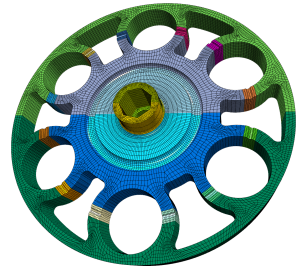
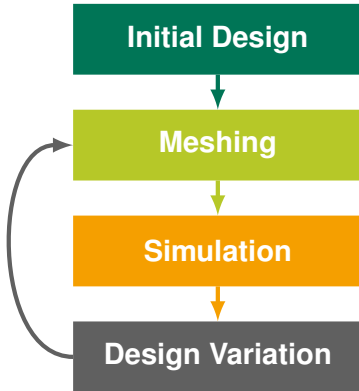
# Design Optimization Scenario



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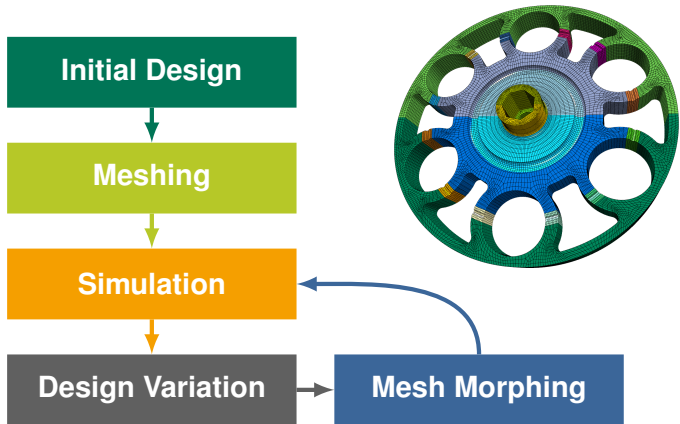


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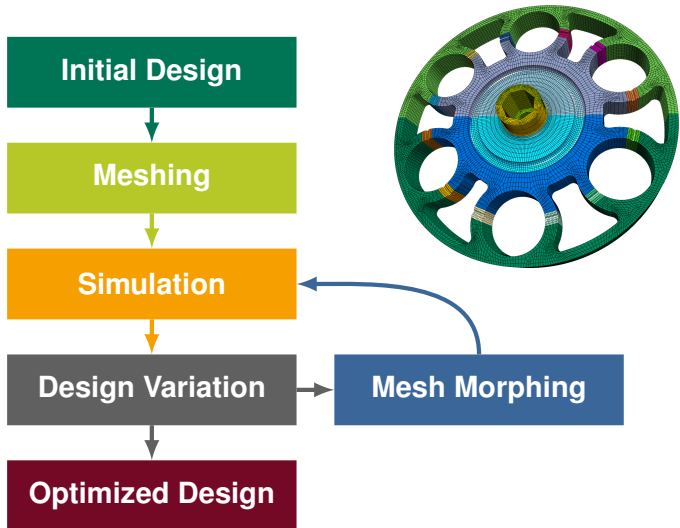




# Design Optimization Scenario



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# Mesh Morphing Methods

- Various methods, significant differences
  - Quality
  - Flexibility
  - Performance

# Mesh Morphing Methods

- Various methods, significant differences
  - Quality
  - Flexibility
  - Performance
- [Staten et al., 2011]: A comparison of mesh morphing methods for 3D shape optimization
- [Sieger et al., 2012]: A comprehensive comparison of shape deformation methods in evolutionary design optimization
- Our related work section

# Mesh Morphing Methods

- Barycentric coordinates (Simplex-linear)

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# Mesh Morphing Methods

- Barycentric coordinates (Simplex-linear)
  - Mesh smoothing (LBWARP)
  - Mesh-based variational methods (FEMWARP)
  - Space deformations
- Our approach: Combine
- Quality of variational methods
  - Flexibility of space deformations

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1. Introduction
- 2. RBF Volume Morphing**
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# Motivation

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→ Radial basis functions (RBFs)

# RBF Space Warp


$$d: \mathbb{R}^3 \rightarrow \mathbb{R}^3$$

# RBF Space Warp

$$\mathbf{d}(\mathbf{p}) = \sum_{j=1}^m \mathbf{w}_j \varphi_j(\mathbf{p}) + \boldsymbol{\pi}(\mathbf{p})$$

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Basis functions at centers  $\mathbf{c}_j$

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Polynomial term

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- Where to place kernels?

→ On the surface nodes

# RBF Morphing

- Determine weights and polynomial coefficients



# RBF Morphing

- Determine weights and polynomial coefficients
- Solve linear system

$$\begin{pmatrix} \Phi & \Pi \\ \Pi^T & \mathbf{0} \end{pmatrix} \begin{pmatrix} \mathbf{W} \\ \mathbf{Q} \end{pmatrix} = \begin{pmatrix} \bar{\mathbf{D}} \\ \mathbf{0} \end{pmatrix}$$

# RBF Morphing

- Determine weights and polynomial coefficients
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The diagram illustrates the linear system for RBF morphing. At the top, a box labeled "Basis function weights" has a downward arrow pointing to the  $\mathbf{W}$  matrix in the equation. At the bottom, a box labeled "Polynomial coefficients" has an upward arrow pointing to the  $\mathbf{Q}$  matrix in the equation. The equation is:

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Basis function weights

$$\begin{pmatrix} \Phi & \Pi \\ \Pi^T & \mathbf{0} \end{pmatrix} \begin{pmatrix} \mathbf{W} \\ \mathbf{Q} \end{pmatrix} = \begin{pmatrix} \bar{\mathbf{D}} \\ \mathbf{0} \end{pmatrix}$$

Polynomial coefficients

# RBF Morphing

- Determine weights and polynomial coefficients

→ Solve linear system

The diagram illustrates the linear system for RBF morphing. It features a central matrix equation with three annotated boxes:

- A green box at the top left contains the formula  $\Phi_{ij} = \varphi_j(\mathbf{p}_i)$ . A green arrow points from this box to the  $\Phi$  block in the top-left corner of the matrix.
- A blue box at the top center contains the text "Basis function weights". A blue arrow points from this box to the  $\mathbf{W}$  block in the top-right corner of the matrix.
- An orange box at the bottom left contains the formula  $\Pi_{ij} = \pi_j(\mathbf{p}_i)$ . An orange arrow points from this box to the  $\Pi^T$  block in the bottom-left corner of the matrix.
- A blue box at the bottom center contains the text "Polynomial coefficients". A blue arrow points from this box to the  $\mathbf{Q}$  block in the bottom-right corner of the matrix.

$$\begin{pmatrix} \Phi & \Pi \\ \Pi^T & \mathbf{0} \end{pmatrix} \begin{pmatrix} \mathbf{W} \\ \mathbf{Q} \end{pmatrix} = \begin{pmatrix} \bar{\mathbf{D}} \\ \mathbf{0} \end{pmatrix}$$

# RBF Morphing

- Determine weights and polynomial coefficients
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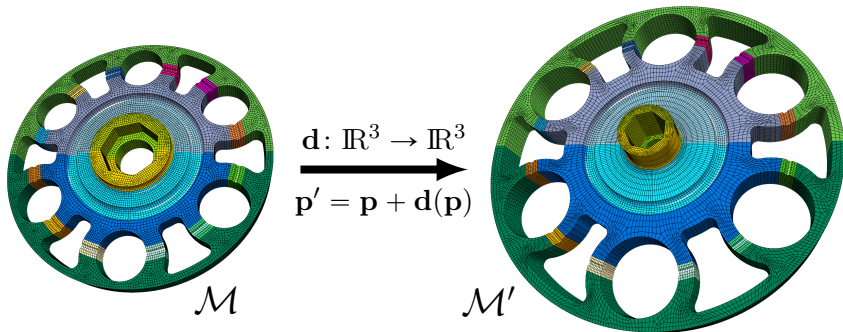
The diagram illustrates the linear system for RBF morphing. It features a central matrix equation with four surrounding boxes and arrows pointing to its components:

- Top-left box (green border):**  $\Phi_{ij} = \varphi_j(\mathbf{p}_i)$ . A green arrow points from this box to the  $\Phi$  block in the matrix.
- Top-middle box (blue border):** Basis function weights. A blue arrow points from this box to the  $\mathbf{W}$  block in the matrix.
- Top-right box (maroon border):** Displacements. A maroon arrow points from this box to the  $\bar{\mathbf{D}}$  block in the matrix.
- Bottom-left box (orange border):**  $\Pi_{ij} = \pi_j(\mathbf{p}_i)$ . An orange arrow points from this box to the  $\Pi^T$  block in the matrix.
- Bottom-middle box (blue border):** Polynomial coefficients. A blue arrow points from this box to the  $\mathbf{Q}$  block in the matrix.

The central equation is:

$$\begin{pmatrix} \Phi & \Pi \\ \Pi^T & \mathbf{0} \end{pmatrix} \begin{pmatrix} \mathbf{W} \\ \mathbf{Q} \end{pmatrix} = \begin{pmatrix} \bar{\mathbf{D}} \\ \mathbf{0} \end{pmatrix}$$

# RBF Morphing

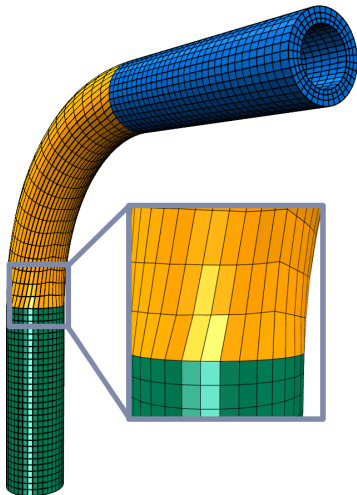


# Surface Quality

*determines* resulting volume mesh quality

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1. Introduction
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# Surface Morphing

- [Staten et al., 2011]: Curve-based morphing
- For each *curve node*  $c_i$  and its curve  $f$  find parameter  $u$  such that  $c_i = f(u)$
- Compute morphed node as  $c'_i = f'(u)$ , where  $f'$  is the morphed curve
- Use  $c'_i$  as input for morphing other surface nodes

# Surface Morphing

- Ideal: *All* surface nodes match modified CAD geometry

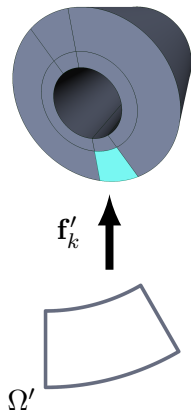
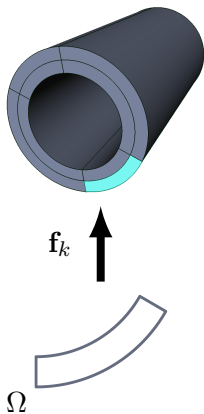
# Surface Morphing

- Ideal: *All* surface nodes match modified CAD geometry
- Extend curve-based approach to *surface nodes* and *faces* so that  $s'_i = \mathbf{f}'(u, v)$

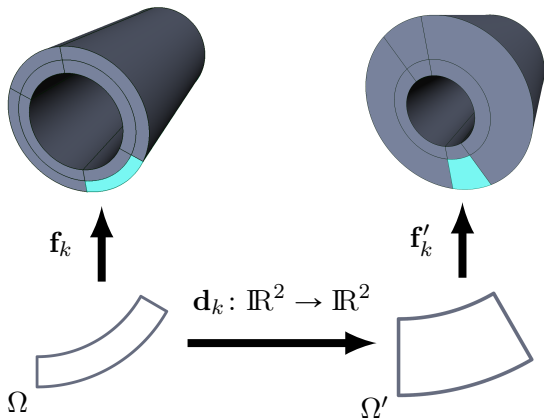
# Surface Morphing

- Ideal: *All* surface nodes match modified CAD geometry
- Extend curve-based approach to *surface nodes* and *faces* so that  $s'_i = \mathbf{f}'(u, v)$
- Problem:  $(u, v)$ -coordinates become invalid due to changed parameter domain and trimming curves

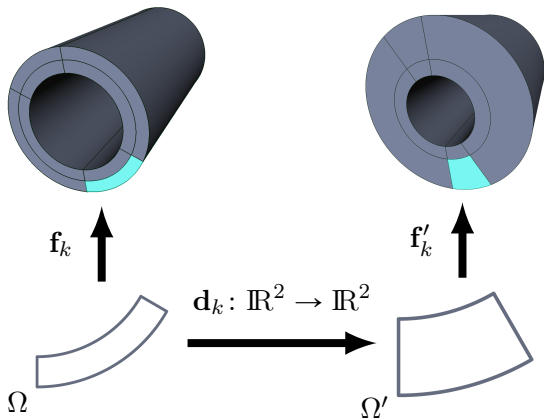
# Surface Morphing



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# Surface Morphing



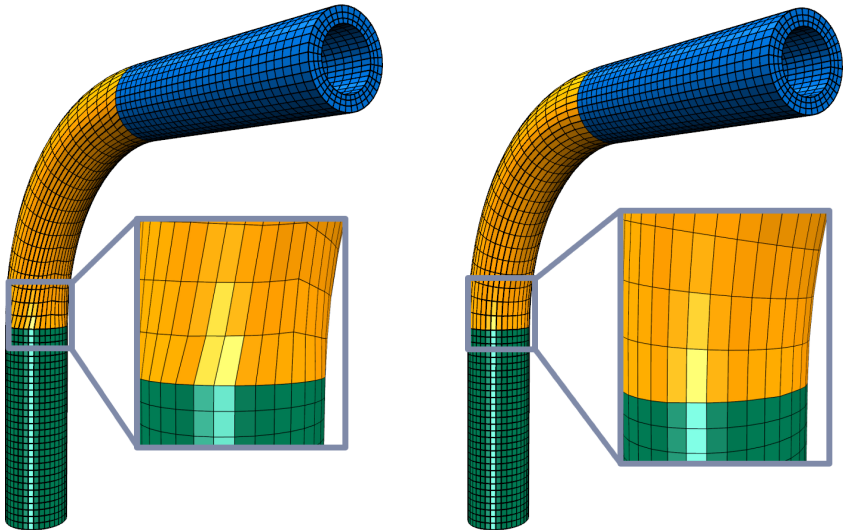
→ Solution: Use RBFs to morph  $(u, v)$  coordinates



# Surface Mesh Morphing

1. Find  $(u, v)$  using dense sampling
2. Construct 2D-triharmonic RBF
  - Sample trimming curve
  - Use samples for centers and displacements
  - Update  $(u', v') = (u, v) + \mathbf{d}_k(u, v)$
3. Evaluate  $\mathbf{f}'(u', v')$  to obtain updated point

# Surface Morphing: Comparison



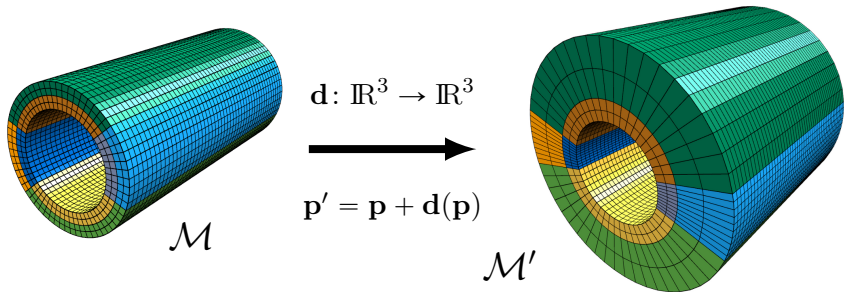
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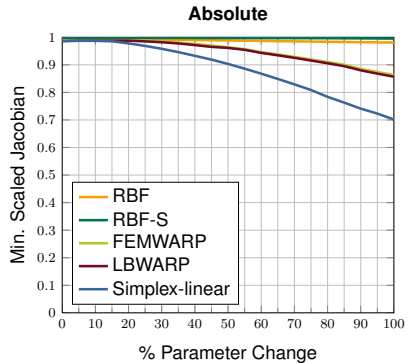
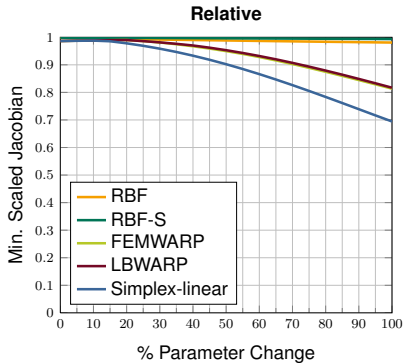
# Benchmarks

- Based on [Staten et al., 2011]
- For each model:
  - Minimum scaled Jacobian vs. % parameter change
  - Iterative and absolute morphing
  - Hex and tet meshes
- Compare our method(s) with
  - FEMWARP
  - LBWARP
  - Simplex-linear

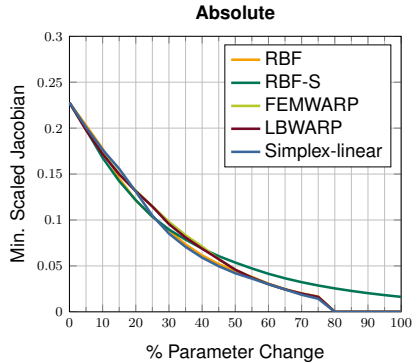
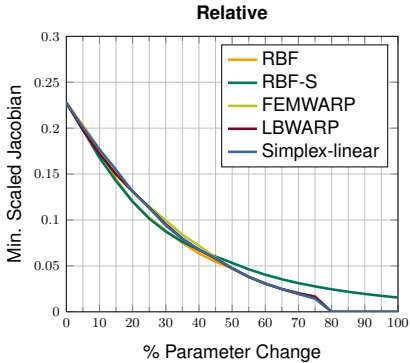
# Bore Model



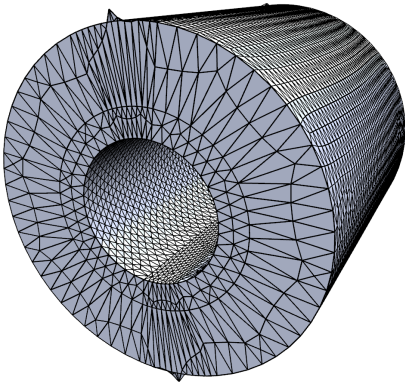
# Bore Hex Model



# Bore Tet Model

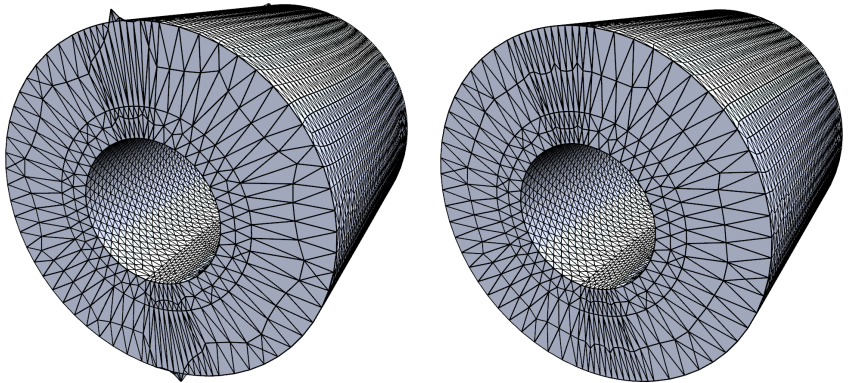


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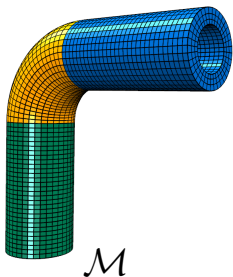




# Bore Tet Model



# Pipe Model

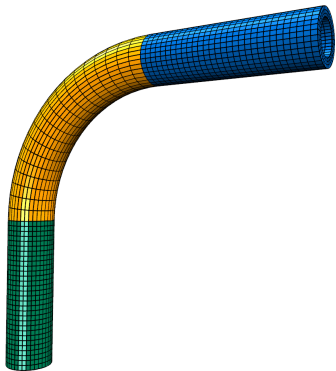


$$\mathbf{d}: \mathbb{R}^3 \rightarrow \mathbb{R}^3$$

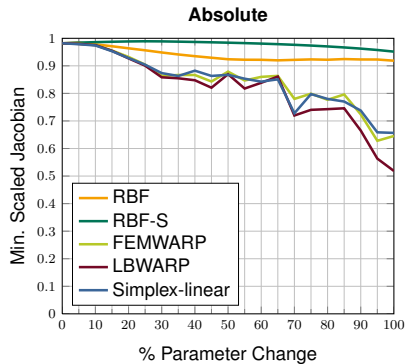
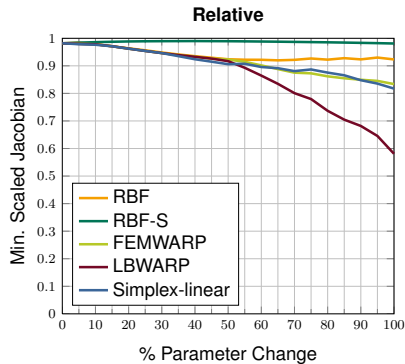
→

$$\mathbf{p}' = \mathbf{p} + \mathbf{d}(\mathbf{p})$$

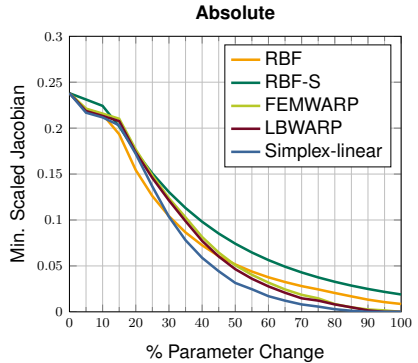
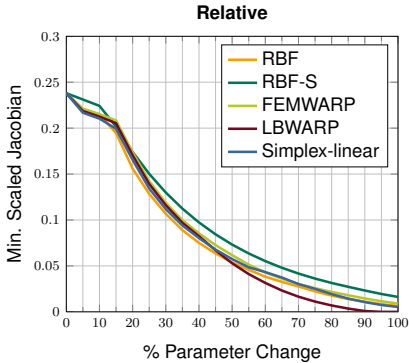
$\mathcal{M}'$



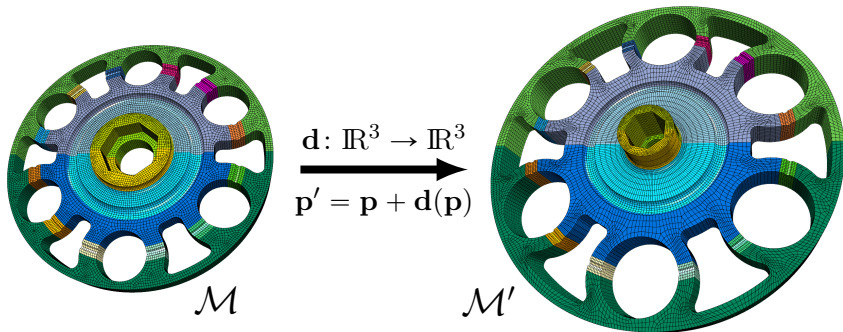
# Pipe Hex Model



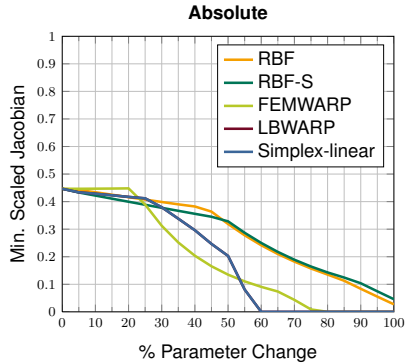
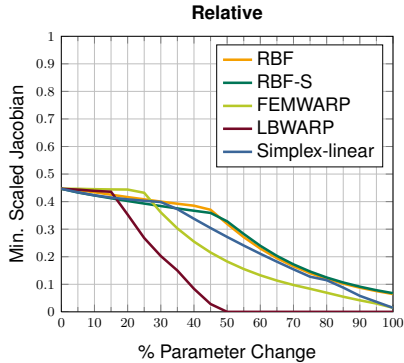
# Pipe Tet Model



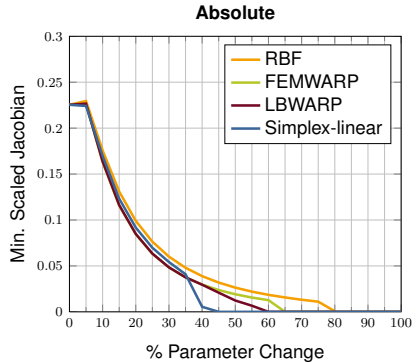
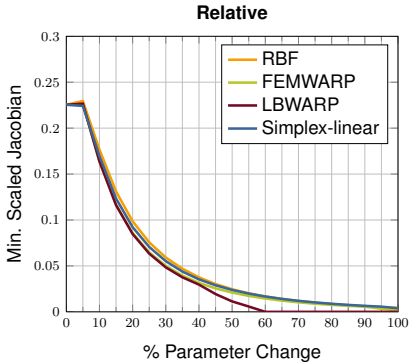
# Courier Model



# Courier Hex Model



# Courier Tet Model



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# Conclusions

- Mesh morphing for simulation-based design optimization
- Triharmonic RBFs
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# Conclusions

- Mesh morphing for simulation-based design optimization
- Triharmonic RBFs
  - Volume meshes
  - Surface morphing
- Our method
  - + Quality
  - + Flexibility
  - + Simplicity
  - Performance

# Acknowledgements

- Many thanks to Matthew Staten for sharing meshes and data
- Honda Research Institute Europe
- German National Research Foundation (DFG CoE 277: CITEC)

# Thanks

... for your attention.

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- Sieger, D., S. Menzel, and M. Botsch (2012). A comprehensive comparison of shape deformation methods in evolutionary design optimization. In *Proceedings of the 3rd International Conference on Engineering Optimization*.
- Staten, M. L., S. J. Owen, S. M. Shontz, A. G. Salinger, and T. S. Coffey (2011). A comparison of mesh morphing methods for 3D shape optimization. In *Proceedings of the 20th International Meshing Roundtable*, pp. 293–311.