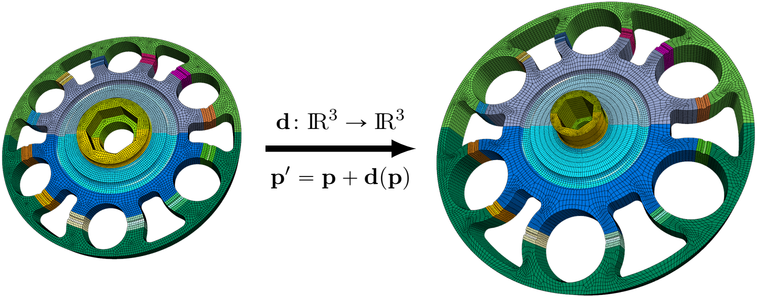
High Quality Mesh Morphing Using Triharmonic Radial Basis Functions

Daniel Sieger, Stefan Menzel, and Mario Botsch



Outline

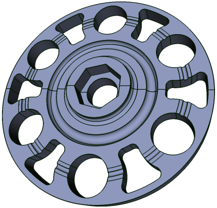
1. **Introduction**
2. RBF Volume Morphing
3. Surface Morphing
4. Results
5. Conclusion

Introduction

- Mesh morphing for simulation-based design optimization
- Update simulation mesh according to updated CAD model

Design Optimization Scenario

Initial Design



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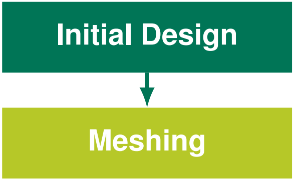
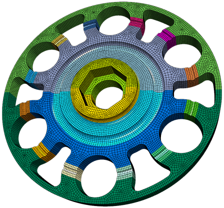




Design Optimization Scenario

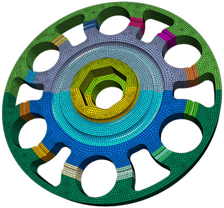
Initial Design

↓

Meshing

↓

Simulation



Design Optimization Scenario

Initial Design
↓
Meshing
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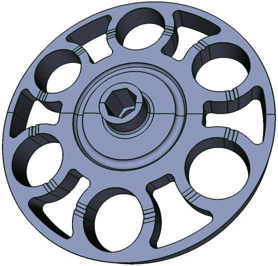
Design Variation



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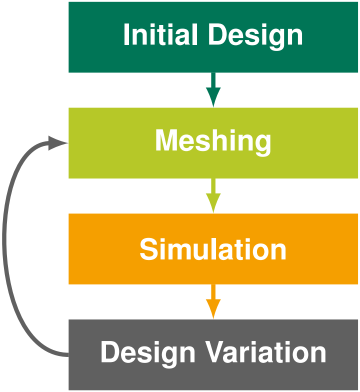

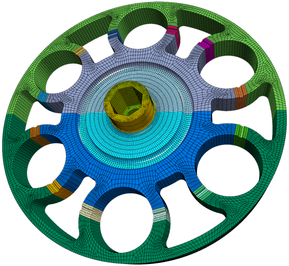

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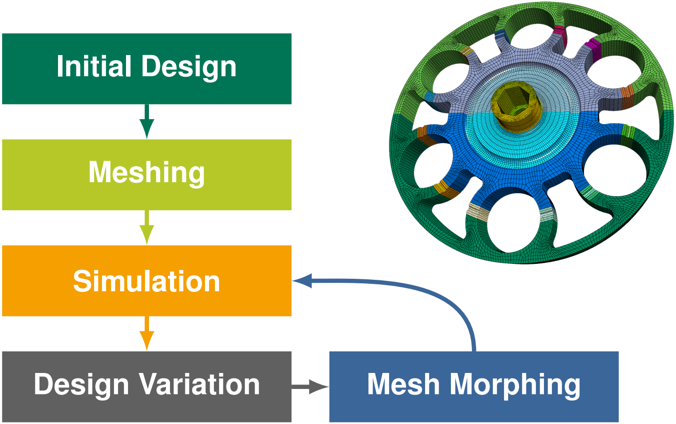

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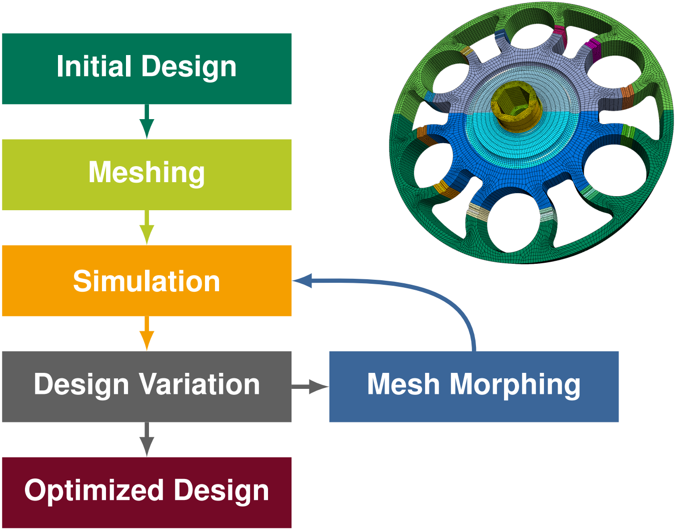

Mesh Morphing Methods

- Various methods, significant differences
  - Quality
  - Flexibility
  - Performance

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  - Flexibility
  - Performance
- [Staten et al., 2011]: A comparison of mesh morphing methods for 3D shape optimization
- [Sieger et al., 2012]: A comprehensive comparison of shape deformation methods in evolutionary design optimization
- Our related work section

Mesh Morphing Methods

- Barycentric coordinates (Simplex-linear)

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➔ Our approach: Combine
  - Quality of variational methods
  - Flexibility of space deformations

Outline

Motivation

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- Morphing as interpolation problem
  - *Exactly* interpolate prescribed displacements
  - *Smoothly* interpolate displacements through space

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➔ Radial basis functions (RBFs)

RBF Space Warp

$$\mathbf{d}\colon \mathbb{R}^3 \rightarrow \mathbb{R}^3$$

RBF Space Warp

$$\mathbf{d}(\mathbf{p}) = \sum_{j=1}^{m} \mathbf{w}_j \varphi_j(\mathbf{p}) + \boldsymbol{\pi}(\mathbf{p})$$

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Basis functions at centers $\mathbf{c}_j$

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- Where to place kernels?

➔ On the surface nodes

RBF Morphing

- Determine weights and polynomial coefficients

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➔ Solve linear system

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Displacements

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Polynomial coefficients

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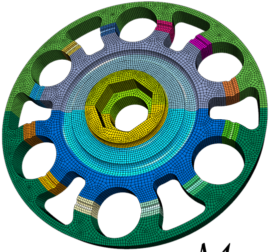

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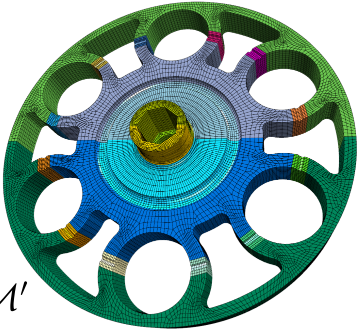

$\mathcal{M}'$

Surface Quality

determines resulting volume mesh quality

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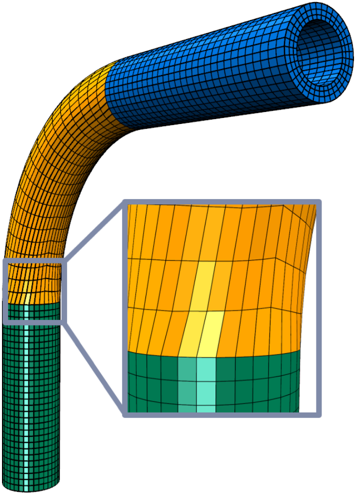

Outline

Surface Morphing

- [Staten et al., 2011]: Curve-based morphing
- For each *curve node* $\mathbf{c}_i$ and its curve $\mathbf{f}$ find parameter u such that $\mathbf{c}_i = \mathbf{f}(u)$
- Compute morphed node as $\mathbf{c}_i' = \mathbf{f}'(u)$, where $\mathbf{f}'$ is the morphed curve
- Use $\mathbf{c}_i'$ as input for morphing other surface nodes

Surface Morphing

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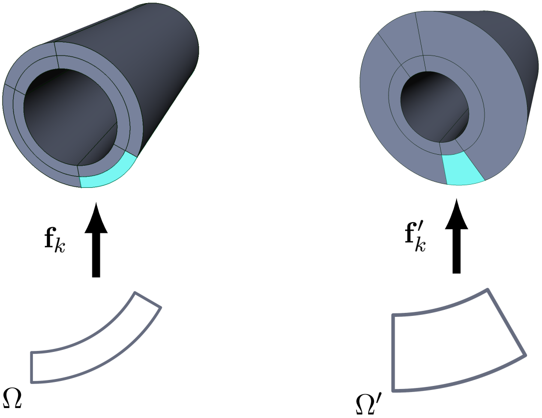
- Extend curve-based approach to *surface nodes* and *faces* so that $\mathrm{s}_i' = \mathbf{f}'(u,v)$
- Problem: (u,v)-coordinates become invalid due to changed parameter domain and trimming curves

Surface Morphing



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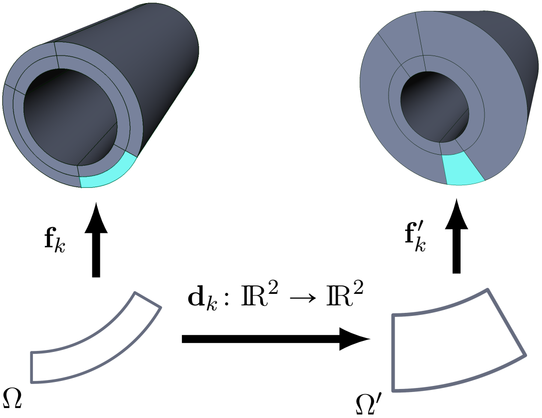

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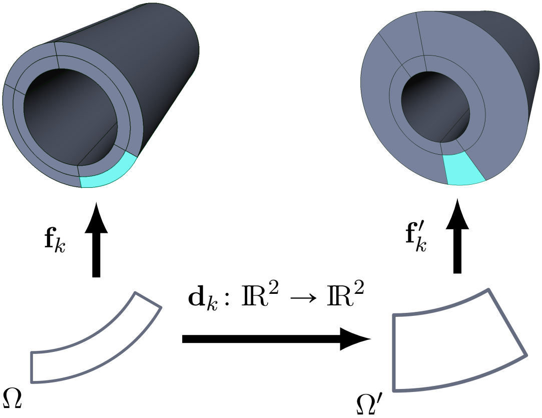


➔ Solution: Use RBFs to morph (u, v) coordinates

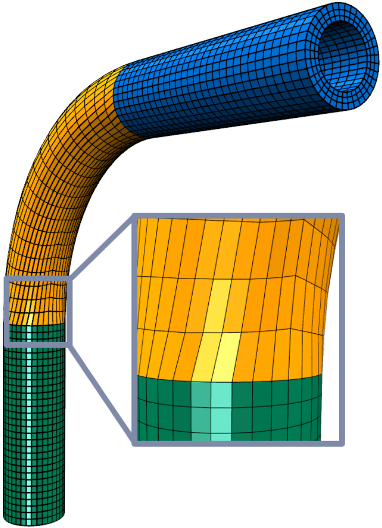
Surface Mesh Morphing

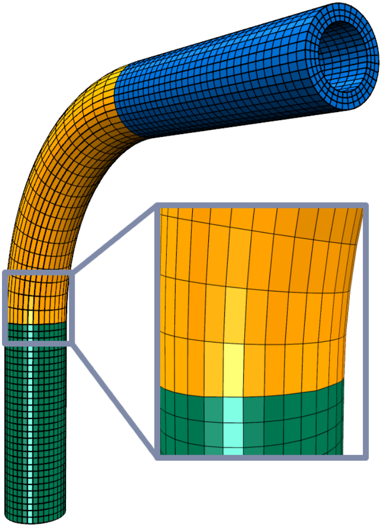
1. Find (u, v) using dense sampling

2. Construct 2D-triharmonic RBF
   - Sample trimming curve
   - Use samples for centers and displacements
   - Update $(u', v') = (u, v) + \mathbf{d}_k(u, v)$

3. Evaluate $\mathbf{f}'(u', v')$ to obtain updated point

Surface Morphing: Comparison





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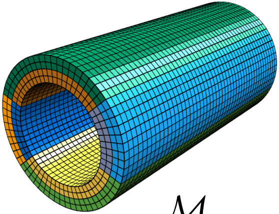
Benchmarks

- Based on [Staten et al., 2011]
- For each model:
  - Minimum scaled Jacobian vs. % parameter change
  - Iterative and absolute morphing
  - Hex and tet meshes
- Compare our method(s) with
  - FEMWARP
  - LBWARP
  - Simplex-linear

Bore Model



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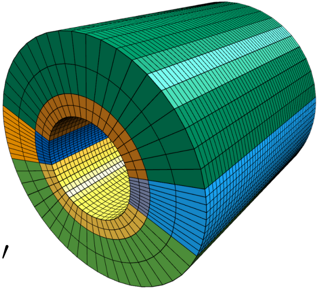
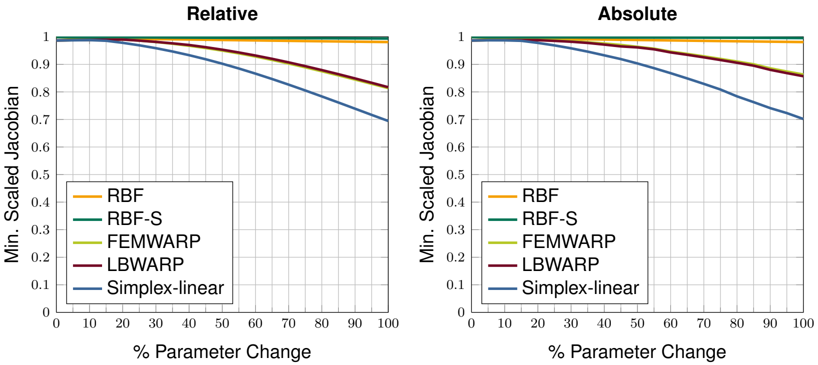
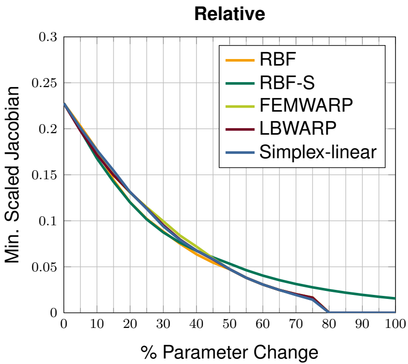


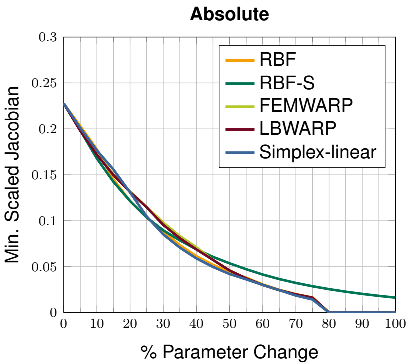
$\mathcal{M}'$

Bore Hex Model



Bore Tet Model





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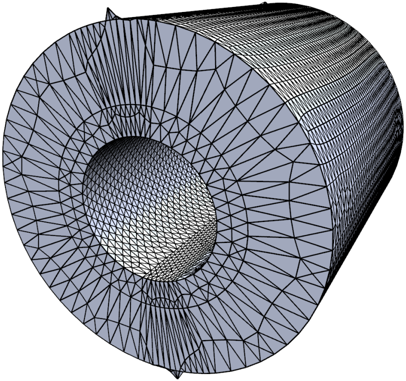

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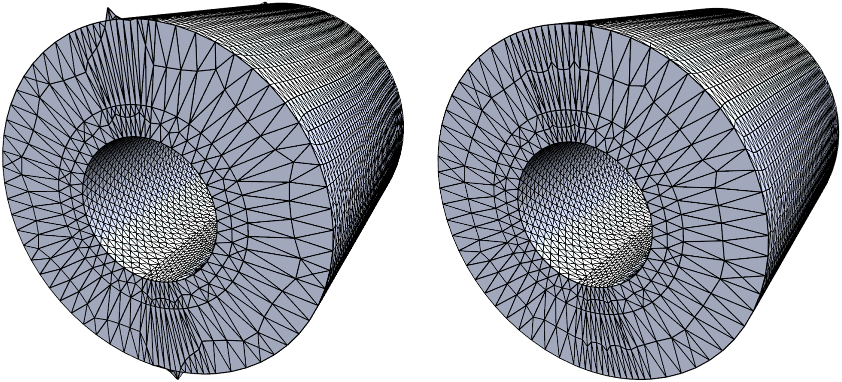
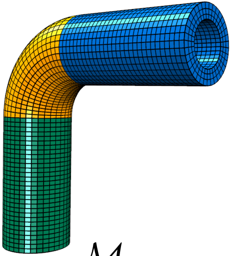


Pipe Model



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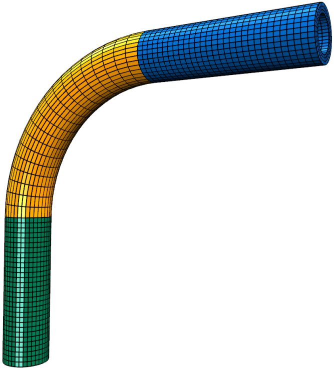

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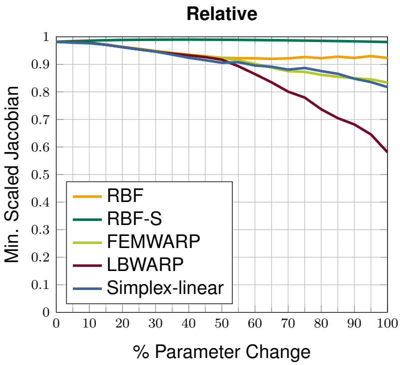

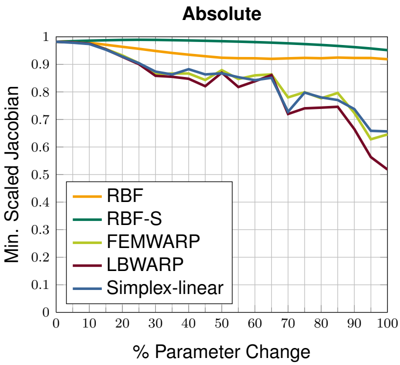

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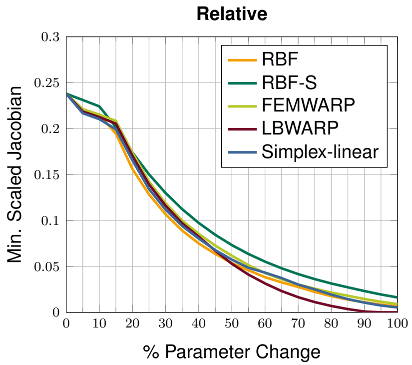
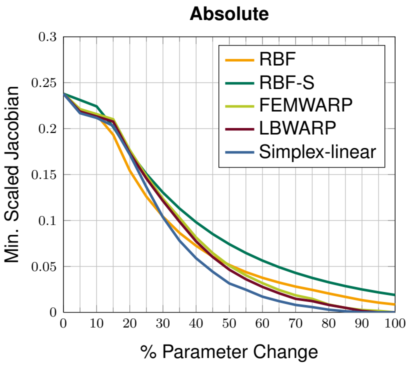
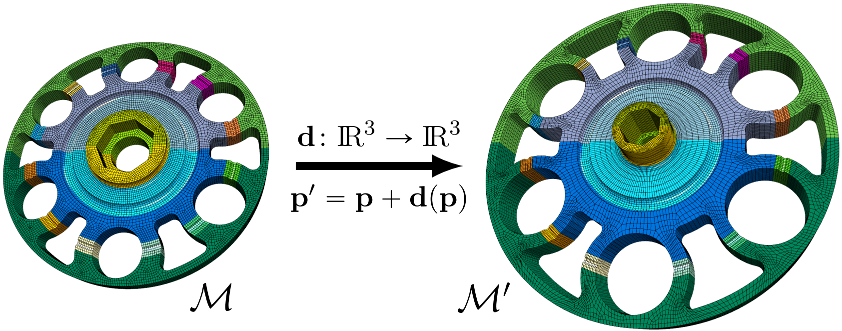
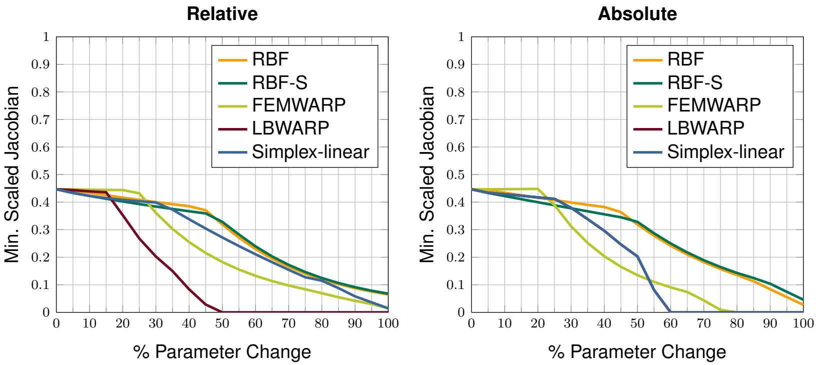




Courier Model



Courier Hex Model



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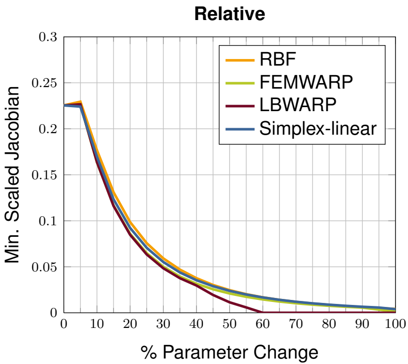

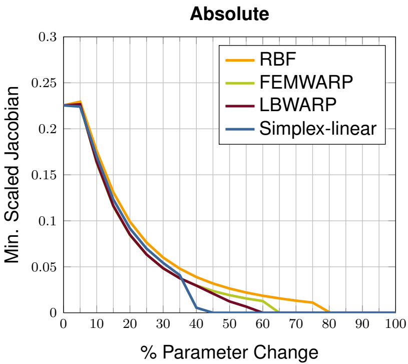

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- Triharmonic RBFs
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- Triharmonic RBFs
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  - Surface morphing
- Our method
  - + Quality
  - + Flexibility
  - + Simplicity
  - – Performance

Acknowledgements

- Many thanks to Matthew Staten for sharing meshes and data
- Honda Research Institute Europe
- German National Research Foundation (DFG CoE 277: CITEC)

Thanks

... for your attention.

References

Sieger, D., S. Menzel, and M. Botsch (2012). A comprehensive comparison of shape deformation methods in evolutionary design optimization. In *Proceedings of the 3rd International Conference on Engineering Optimization*.

Staten, M. L., S. J. Owen, S. M. Shontz, A. G. Salinger, and T. S. Coffey (2011). A comparison of mesh morphing methods for 3D shape optimization. In *Proceedings of the 20th International Meshing Roundtable*, pp. 293–311.