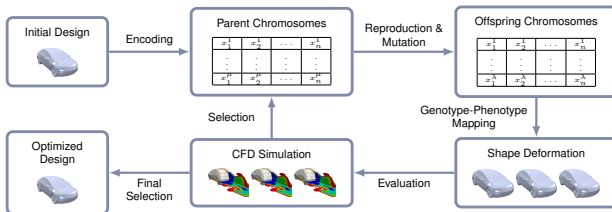


A Comprehensive Comparison of Shape Deformation Methods in Evolutionary Design Optimization



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Outline

1. Shape deformation methods
2. Evolutionary design optimization
3. Application: Passenger car design optimization

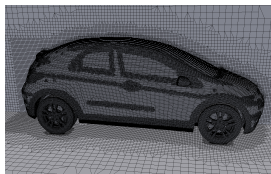
Shape Deformation Methods

Shape Deformation Methods

- Fundamental requirements?

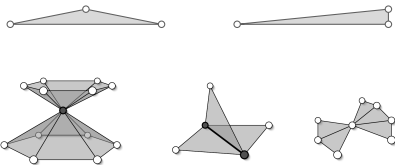
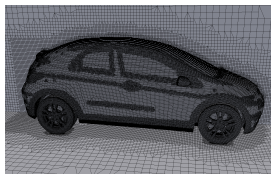
Shape Deformation Methods

- Fundamental requirements?
- Different representations
 - Surface meshes
 - Volume meshes



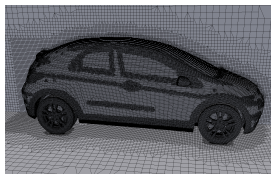
Shape Deformation Methods

- Fundamental requirements?
- Different representations
 - Surface meshes
 - Volume meshes
- Defects
 - Badly shaped elements
 - Non-manifold meshes
 - Self-intersections

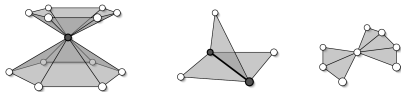


Shape Deformation Methods

- Fundamental requirements?
- Different representations
 - Surface meshes
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→ *Space deformations*




Space Deformation Methods

- Deformation function $d: \mathbb{R}^3 \rightarrow \mathbb{R}^3$
- Warp embedding space around object \mathcal{M}



\mathcal{M}

$$d: \mathbb{R}^3 \rightarrow \mathbb{R}^3$$

$$\mathbf{p}' = \mathbf{p} + d(\mathbf{p})$$




\mathcal{M}'

Space Deformation Methods

- Deformation function $\mathbf{d}: \mathbb{R}^3 \rightarrow \mathbb{R}^3$
- Warp embedding space around object \mathcal{M}
- Methods:
 - Free-form deformation (FFD)
 - Direct manipulation FFD
 - Radial basis functions



\mathcal{M}

$$\mathbf{d}: \mathbb{R}^3 \rightarrow \mathbb{R}^3$$

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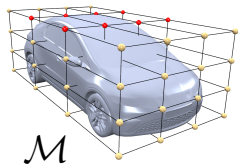


\mathcal{M}'

Free-Form Deformation

Free-Form Deformation (FFD)

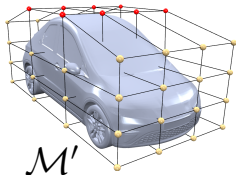
- Embed object in control lattice
- Compute local coordinates
- Move control points
- Deform object according to updated control points



$$\mathbf{d}_{\text{ffd}} : \mathbb{R}^3 \rightarrow \mathbb{R}^3$$

→

$$\mathbf{p}' = \mathbf{p} + \mathbf{d}_{\text{ffd}}(\mathbf{u})$$




Free-Form Deformation: Embedding

$$\mathbf{P} = \sum_{i=0}^l \sum_{j=0}^m \sum_{k=0}^n \mathbf{c}_{ijk} N_i(u_1) N_j(u_2) N_k(u_3)$$

Free-Form Deformation: Embedding

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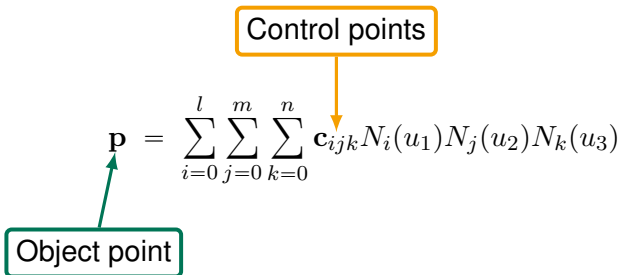
Object point

Free-Form Deformation: Embedding

Control points

$$\mathbf{P} = \sum_{i=0}^l \sum_{j=0}^m \sum_{k=0}^n \mathbf{c}_{ijk} N_i(u_1) N_j(u_2) N_k(u_3)$$

Object point



Free-Form Deformation: Embedding

The diagram illustrates the embedding of control points into the Free-Form Deformation (FFD) equation. The equation is
$$\mathbf{p} = \sum_{i=0}^l \sum_{j=0}^m \sum_{k=0}^n \mathbf{c}_{ijk} N_i(u_1) N_j(u_2) N_k(u_3)$$
 where \mathbf{p} is the object point, \mathbf{c}_{ijk} are the control points, and N_i, N_j, N_k are the basis functions. The control points are highlighted in an orange box, and the object point and basis functions are highlighted in green and blue boxes respectively. Arrows indicate the mapping from the boxes to the corresponding terms in the equation.

Control points

Object point

Basis functions

$$\mathbf{p} = \sum_{i=0}^l \sum_{j=0}^m \sum_{k=0}^n \mathbf{c}_{ijk} N_i(u_1) N_j(u_2) N_k(u_3)$$

Free-Form Deformation: Embedding

The diagram illustrates the embedding of control points and local coordinates into the Free-Form Deformation (FFD) equation. The equation is:

$$\mathbf{P} = \sum_{i=0}^l \sum_{j=0}^m \sum_{k=0}^n \mathbf{c}_{ijk} N_i(u_1) N_j(u_2) N_k(u_3)$$

The components are labeled as follows:

- Object point**: Points to the vector \mathbf{P} .
- Control points**: Points to the coefficient \mathbf{c}_{ijk} .
- Basis functions**: Points to the product of basis functions $N_i(u_1) N_j(u_2) N_k(u_3)$.
- Local coordinates**: Points to the variables u_1, u_2, u_3 within the basis functions.

Free-Form Deformation Function

$$\mathbf{d}_{\text{ffd}}(\mathbf{u}) = \sum_p \delta \mathbf{c}_p N_p(\mathbf{u})$$

Free-Form Deformation Function

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Local coordinates

$$\mathbf{u} := (u_1, u_2, u_3)$$

Free-Form Deformation Function

Control point displacements

$$\delta \mathbf{c}_p := \delta \mathbf{c}_{ijk} = \mathbf{c}'_{ijk} - \mathbf{c}_{ijk}$$

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Basis functions

$$N_p(\mathbf{u}) := N_i(u_1)N_j(u_2)N_k(u_3)$$

Free-Form Deformation: Caveats

- Difficult control grid generation
- Numerical coordinate computation
- Tedious control point manipulation
- Only indirect influence on the shape

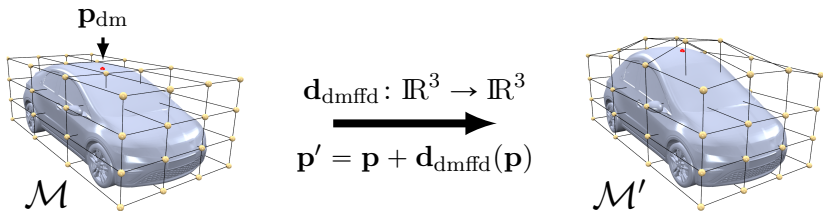
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Direct Manipulation FFD

Direct Manipulation FFD

- Move object points directly
- Automatically compute control point displacements satisfying new object point locations



Direct Manipulation FFD

- Solve linear system to compute control point displacements:

$$\begin{bmatrix} N_1(\mathbf{u}_1) & \dots & N_n(\mathbf{u}_1) \\ \vdots & \ddots & \vdots \\ N_1(\mathbf{u}_m) & \dots & N_n(\mathbf{u}_m) \end{bmatrix} \begin{pmatrix} \delta \mathbf{c}_1^T \\ \vdots \\ \delta \mathbf{c}_n^T \end{pmatrix} = \begin{pmatrix} \bar{\mathbf{d}}_1^T \\ \vdots \\ \bar{\mathbf{d}}_m^T \end{pmatrix}.$$

- System is singular \rightarrow solve using pseudo-inverse

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 - *Exactly* interpolate prescribed displacements
 - *Smoothly* interpolate displacements through space

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Basis functions at centers \mathbf{c}_j

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→ Radial basis functions (RBFs)

The diagram illustrates the components of the RBF deformation equation. A green box at the top contains the text "Basis functions at centers c_j ". A green arrow points from this box down to the term $\varphi_j(\mathbf{p})$ in the equation below. An orange box at the bottom contains the text "Weights". An orange arrow points from this box up to the term \mathbf{w}_j in the equation. The equation is
$$\mathbf{d}_{\text{rbf}}(\mathbf{p}) = \sum_{j=1}^m \mathbf{w}_j \varphi_j(\mathbf{p}) + \boldsymbol{\pi}(\mathbf{p})$$

Radial Basis Function Deformation

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→ Radial basis functions (RBFs)

The diagram illustrates the equation for radial basis function deformation. At the top, a green-bordered box contains the text "Basis functions at centers c_j ". A green arrow points from this box down to the $\varphi_j(\mathbf{p})$ term in the equation. Below the equation, an orange-bordered box labeled "Weights" has an orange arrow pointing up to the \mathbf{w}_j term. To the right, a blue-bordered box labeled "Polynomial term" has a blue arrow pointing up to the $\pi(\mathbf{p})$ term. The equation itself is
$$\mathbf{d}_{\text{rbf}}(\mathbf{p}) = \sum_{j=1}^m \mathbf{w}_j \varphi_j(\mathbf{p}) + \pi(\mathbf{p})$$

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$$\mathbf{d}_{\text{rbf}}(\mathbf{p}) = \sum_{j=1}^m \mathbf{w}_j \varphi_j(\mathbf{p}) + \pi(\mathbf{p})$$

Radial Basis Functions

- Various choices: Gaussian, multiquadrics, thin plate spline...

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- Choose $\varphi(r) = r^3$ so that \mathbf{d} minimizes fairness energy:

$$\int_{\mathbb{R}^3} \left\| \frac{\partial^3 \mathbf{d}}{\partial x^3} \right\|^2 + \left\| \frac{\partial^3 \mathbf{d}}{\partial x^2 \partial y} \right\|^2 + \dots + \left\| \frac{\partial^3 \mathbf{d}}{\partial z^3} \right\|^2 dV.$$

Radial Basis Functions

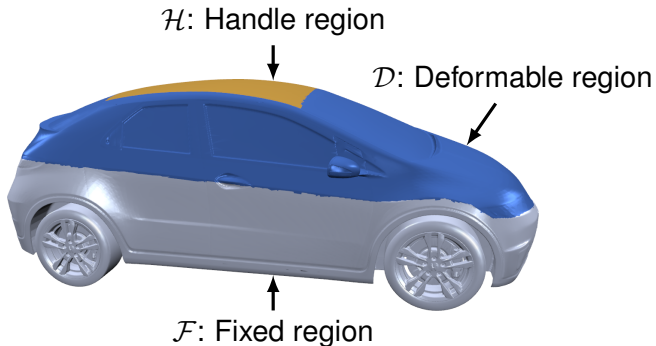
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- Where to place kernels?

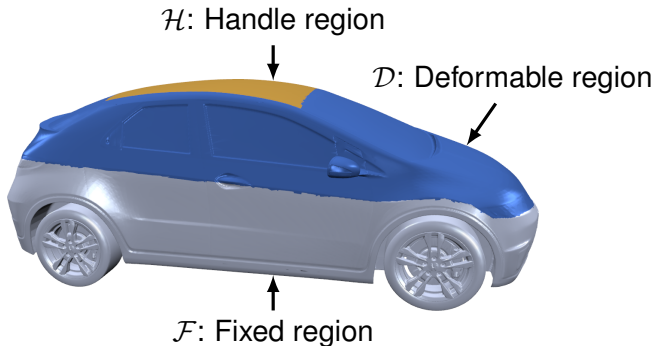
Radial Basis Function Deformation

- Handle-based direct manipulation interface



Radial Basis Function Deformation

- Handle-based direct manipulation interface
- Place kernels in handle and fixed regions



Radial Basis Function Deformation

- Determine weights and polynomial coefficients

Radial Basis Function Deformation

- Determine weights and polynomial coefficients
- Solve linear system

$$\begin{pmatrix} \Phi & \Pi \\ \Pi^T & \mathbf{0} \end{pmatrix} \begin{pmatrix} \mathbf{W} \\ \mathbf{Q} \end{pmatrix} = \begin{pmatrix} \bar{\mathbf{D}} \\ \mathbf{0} \end{pmatrix}$$

Radial Basis Function Deformation

- Determine weights and polynomial coefficients

→ Solve linear system

The diagram illustrates the relationship between basis function weights and polynomial coefficients in a linear system. At the top, a box labeled "Basis function weights" has a downward arrow pointing to the \mathbf{W} component of the vector $\begin{pmatrix} \mathbf{W} \\ \mathbf{Q} \end{pmatrix}$ in the matrix equation. At the bottom, a box labeled "Polynomial coefficients" has an upward arrow pointing to the \mathbf{Q} component of the same vector. The matrix equation is:

$$\begin{pmatrix} \Phi & \Pi \\ \Pi^T & \mathbf{0} \end{pmatrix} \begin{pmatrix} \mathbf{W} \\ \mathbf{Q} \end{pmatrix} = \begin{pmatrix} \bar{\mathbf{D}} \\ \mathbf{0} \end{pmatrix}$$

Radial Basis Function Deformation

- Determine weights and polynomial coefficients

→ Solve linear system

$$\Phi_{ij} = \varphi_j(\mathbf{p}_i)$$

Basis function weights

$$\begin{pmatrix} \Phi & \Pi \\ \Pi^T & \mathbf{0} \end{pmatrix} \begin{pmatrix} \mathbf{W} \\ \mathbf{Q} \end{pmatrix} = \begin{pmatrix} \bar{\mathbf{D}} \\ \mathbf{0} \end{pmatrix}$$

Polynomial coefficients

Radial Basis Function Deformation

- Determine weights and polynomial coefficients

→ Solve linear system

The diagram illustrates the linear system for RBF deformation. It features a central matrix equation with four annotated boxes:

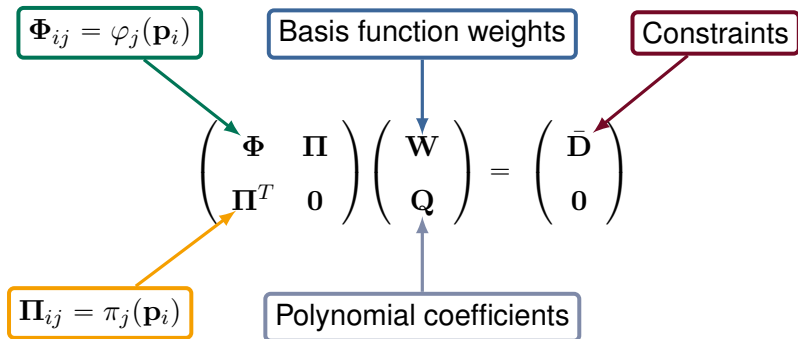
- Green box:** $\Phi_{ij} = \varphi_j(\mathbf{p}_i)$ with an arrow pointing to the Φ block in the matrix.
- Blue box (top):** Basis function weights with an arrow pointing to the \mathbf{W} block in the vector.
- Blue box (bottom):** Polynomial coefficients with an arrow pointing to the \mathbf{Q} block in the vector.
- Orange box:** $\Pi_{ij} = \pi_j(\mathbf{p}_i)$ with an arrow pointing to the Π^T block in the matrix.

$$\begin{pmatrix} \Phi & \Pi \\ \Pi^T & \mathbf{0} \end{pmatrix} \begin{pmatrix} \mathbf{W} \\ \mathbf{Q} \end{pmatrix} = \begin{pmatrix} \bar{\mathbf{D}} \\ \mathbf{0} \end{pmatrix}$$

Radial Basis Function Deformation

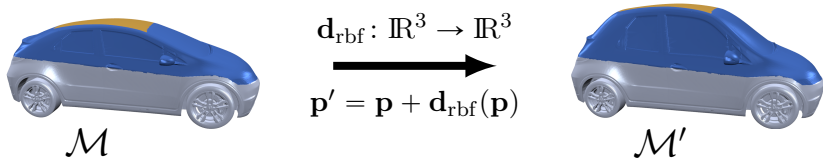
- Determine weights and polynomial coefficients

→ Solve linear system



RBF Deformation

- + Smooth and physically plausible
- + Satisfies constraints exactly
- + No control lattice, flexible setup



Evolutionary Design Optimization

Evolutionary Algorithms

- + Global optimization
- + Generate novel designs
- + Robustness to noise
- + Non-smooth, multi-objective target functions

Evolutionary Algorithms

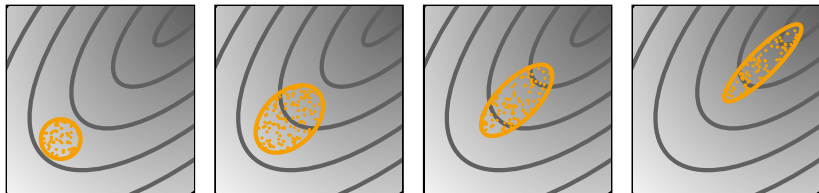
- + Global optimization
- + Generate novel designs
- + Robustness to noise
- + Non-smooth, multi-objective target functions
- Computationally expensive

Evolution Strategies (ES)

- Represent solutions as vectors of real numbers
- Create offspring by adding zero mean random vector
- Advantages:
 - + Self-adaptation of strategy parameters during optimization
 - + Simple incorporation of constraints

Covariance Matrix Adaptation ES

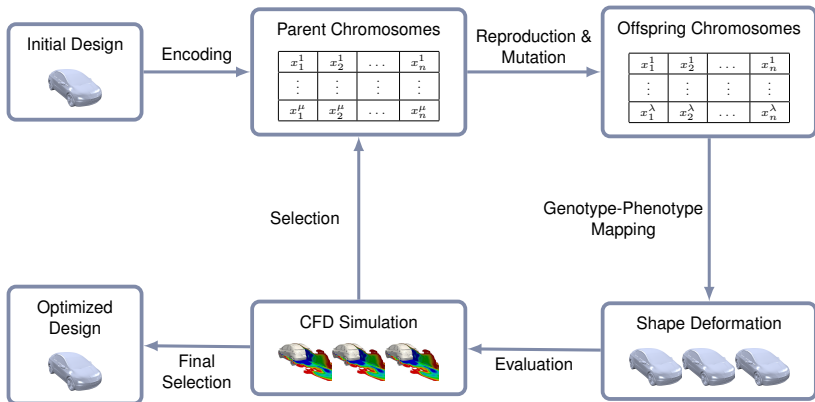
- Adapt covariance matrix to previously successful solutions
- + Fast convergence on small population sizes



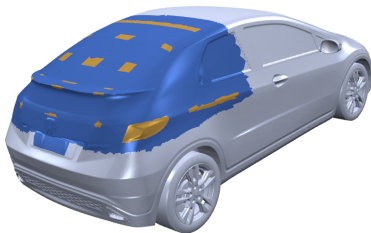
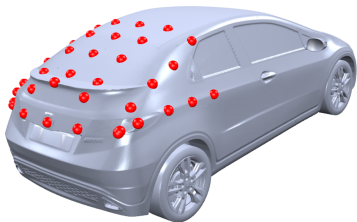
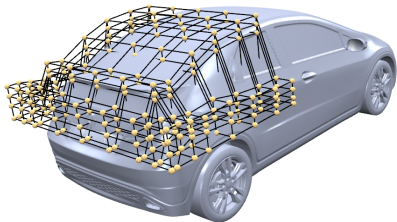
Passenger Car Design Optimization

Passenger Car Design Optimization

Goal: Improve aerodynamic drag of a simplified Honda Civic



Deformation Setups



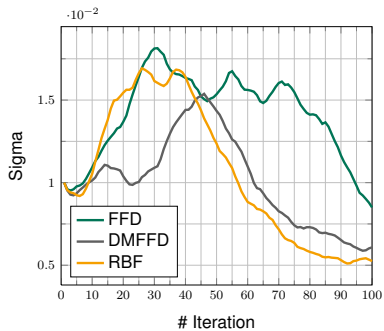
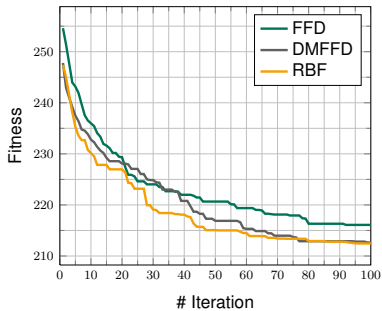
Fitness Function Evaluation

- Fitness function $f: \mathbb{R}^2 \rightarrow \mathbb{R}$:

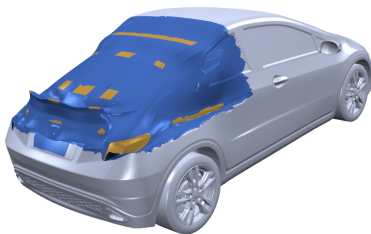
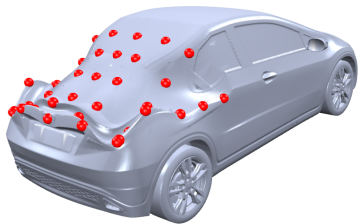
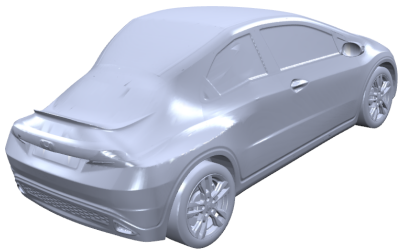
$$f(\mathbf{x}) = w_1 v_1 + w_2 v_2.$$

- v_1 : aerodynamic drag computed by CFD simulation
- v_2 : volume weight to penalize overly flat shapes

Results



Results



Conclusions & Future Work

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- Conclusions:
 - RBFs: Flexible setup with equivalent or better results
 - Strong coupling is important

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- Conclusions:
 - RBFs: Flexible setup with equivalent or better results
 - Strong coupling is important
- Future work:
 - Additional methods
 - Unified interface
 - Synthetic benchmarks

Thanks

... for your attention.

Questions?