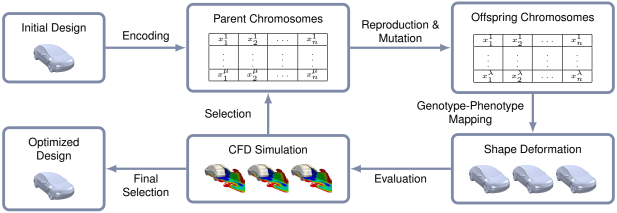
A Comprehensive Comparison of Shape Deformation Methods in Evolutionary Design Optimization



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[2] Honda Research Institute

Universität Bielefeld

Honda Research Institute EU

CITEC

Outline

1. Shape deformation methods
2. Evolutionary design optimization
3. Application: Passenger car design optimization

Shape Deformation Methods

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- Fundamental requirements?

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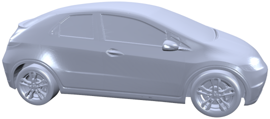
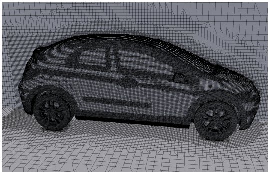
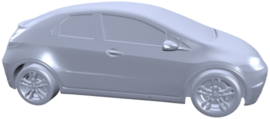
- Different representations
  - Surface meshes
  - Volume meshes

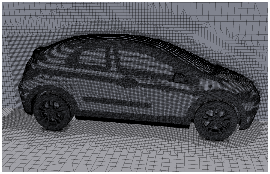


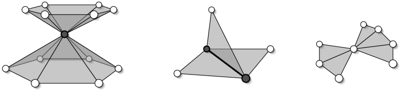


Shape Deformation Methods

- Fundamental requirements?
- Different representations
  - Surface meshes
  - Volume meshes
- Defects
  - Badly shaped elements
  - Non-manifold meshes
  - Self-intersections







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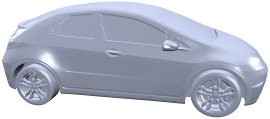

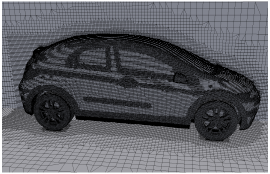

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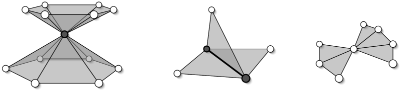
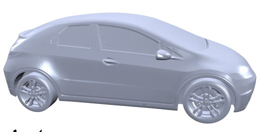


→ *Space deformations*

Space Deformation Methods

- Deformation function $\mathbf{d}\colon \mathbb{R}^3 \to \mathbb{R}^3$
- Warp embedding space around object $\mathcal{M}$



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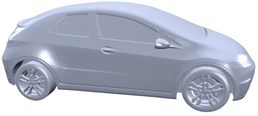
$$\mathbf{d}\colon \mathbb{R}^3 \to \mathbb{R}^3$$

$$\mathbf{p}' = \mathbf{p} + \mathbf{d}(\mathbf{p})$$

$\mathcal{M}'$

Space Deformation Methods

- Deformation function $\mathbf{d}\colon \mathbb{R}^3 \to \mathbb{R}^3$
- Warp embedding space around object $\mathcal{M}$
- Methods:
  - Free-form deformation (FFD)
  - Direct manipulation FFD
  - Radial basis functions



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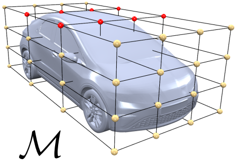
Free-Form Deformation

Free-Form Deformation (FFD)

- Embed object in control lattice
- Compute local coordinates
- Move control points
- Deform object according to updated control points



$$\mathbf{d}_{\text{ffd}} : \mathbb{R}^3 \rightarrow \mathbb{R}^3$$

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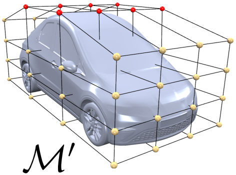

Free-Form Deformation: Embedding

$$\mathbf{p} = \sum_{i=0}^{l} \sum_{j=0}^{m} \sum_{k=0}^{n} \mathbf{c}_{ijk} N_i(u_1) N_j(u_2) N_k(u_3)$$

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Local coordinates
$\mathbf{u} := (u_1, u_2, u_3)$

Free-Form Deformation Function

Control point displacements
$\delta \mathbf{c}_p := \delta \mathbf{c}_{ijk} = \mathbf{c}'_{ijk} - \mathbf{c}_{ijk}$

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$$N_p(\mathbf{u}) := N_i(u_1) N_j(u_2) N_k(u_3)$$

Free-Form Deformation: Caveats

- Difficult control grid generation
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Direct Manipulation FFD

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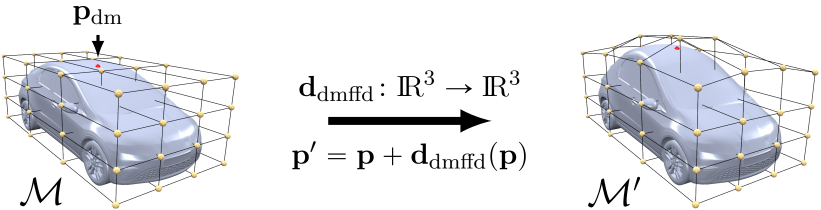
- Move object points directly
- Automatically compute control point displacements satisfying new object point locations



Direct Manipulation FFD

- Solve linear system to compute control point displacements:

$$\begin{bmatrix} N_1(\mathbf{u}_1) & \dots & N_n(\mathbf{u}_1) \\ \vdots & \ddots & \vdots \\ N_1(\mathbf{u}_m) & \dots & N_n(\mathbf{u}_m) \end{bmatrix} \begin{pmatrix} \delta\mathbf{c}_1^T \\ \vdots \\ \delta\mathbf{c}_n^T \end{pmatrix} = \begin{pmatrix} \bar{\mathbf{d}}_1^T \\ \vdots \\ \bar{\mathbf{d}}_m^T \end{pmatrix}.$$

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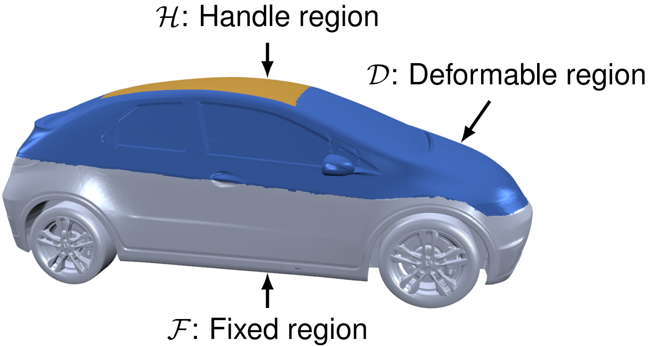
- Where to place kernels?

Radial Basis Function Deformation

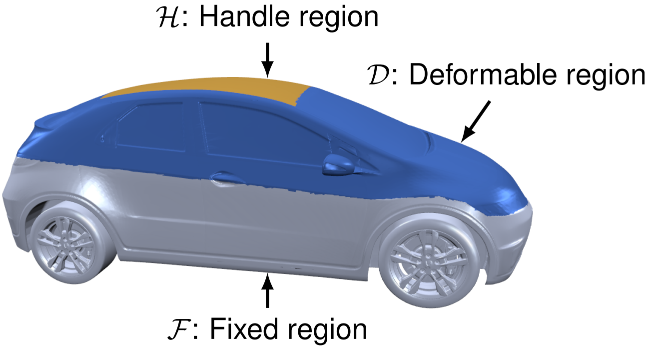
- Handle-based direct manipulation interface



Radial Basis Function Deformation

- Handle-based direct manipulation interface

→ Place kernels in handle and fixed regions



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→ Solve linear system

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Constraints

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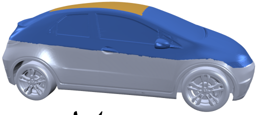
Polynomial coefficients

RBF Deformation

- + Smooth and physically plausible
- + Satisfies constraints exactly
- + No control lattice, flexible setup



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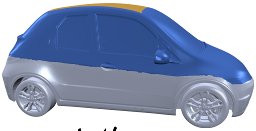

$\mathcal{M}'$

Evolutionary Design Optimization

Evolutionary Algorithms

+ Global optimization
+ Generate novel designs
+ Robustness to noise
+ Non-smooth, multi-objective target functions

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- + Global optimization
- + Generate novel designs
- + Robustness to noise
- + Non-smooth, multi-objective target functions
- – Computationally expensive

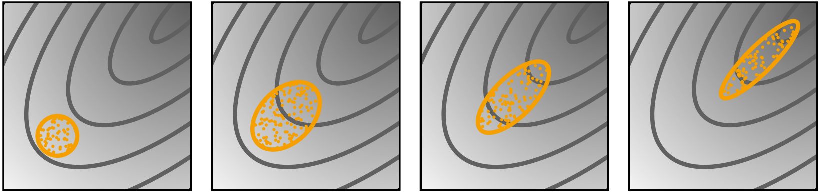
Evolution Strategies (ES)

- Represent solutions as vectors of real numbers
- Create offspring by adding zero mean random vector
- Advantages:
  - + Self-adaptation of strategy parameters during optimization
  - + Simple incorporation of constraints

Covariance Matrix Adaptation ES

- Adapt covariance matrix to previously successful solutions

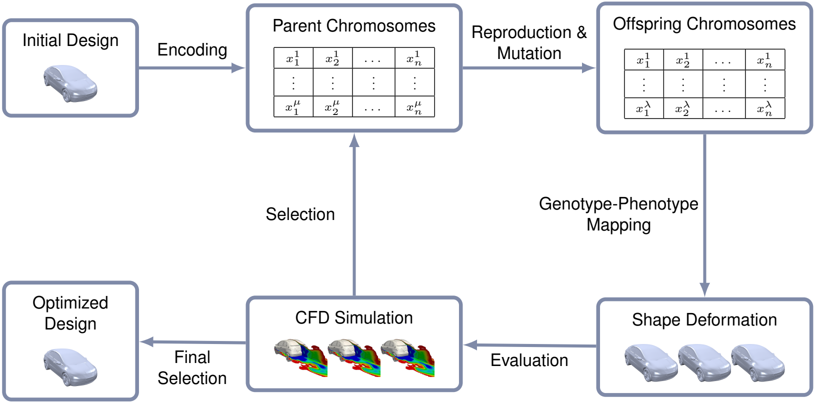
+ Fast convergence on small population sizes



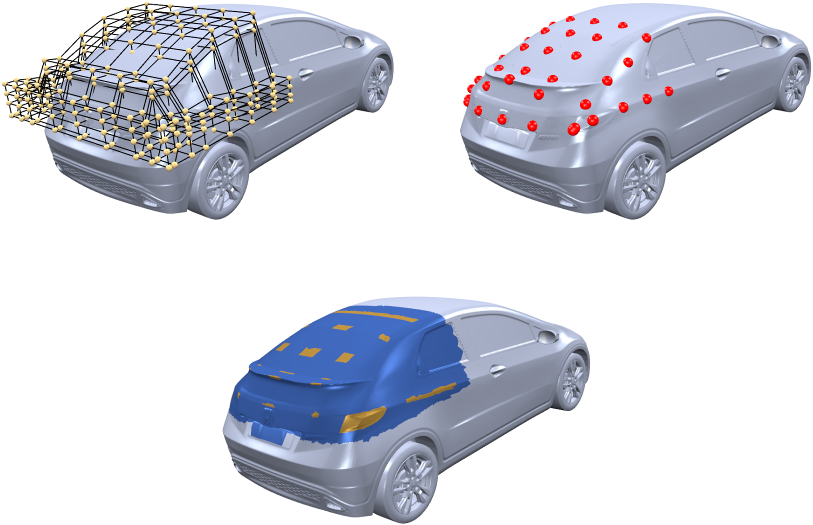
Passenger Car
Design Optimization

Passenger Car Design Optimization

Goal: Improve aerodynamic drag of a simplified Honda Civic



Deformation Setups



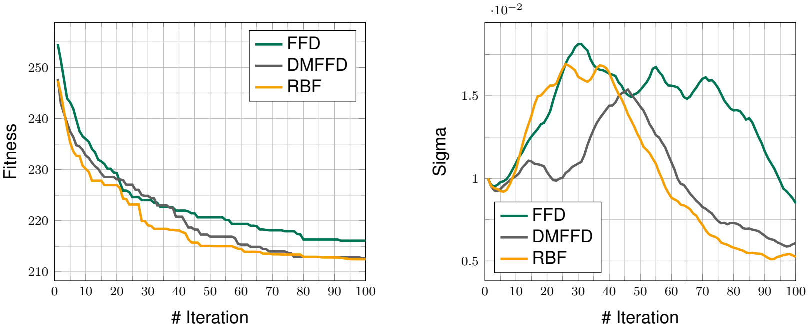
Fitness Function Evaluation

- Fitness function $f\colon \mathbb{R}^2 \to \mathbb{R}$:

$$f(\mathbf{x}) = w_1 v_1 + w_2 v_2.$$

- v_1: aerodynamic drag computed by CFD simulation
- v_2: volume weight to penalize overly flat shapes

Results



FFD
DMFFD
RBF
Fitness
250
240
230
220
210
0
10
20
30
40
50
60
70
80
90
100
Iteration
·10^−2
Sigma
1.5
1
0.5
FFD
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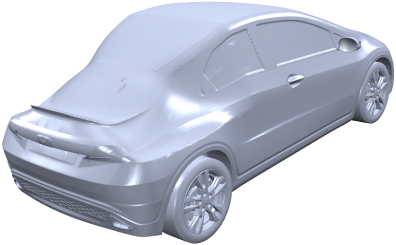

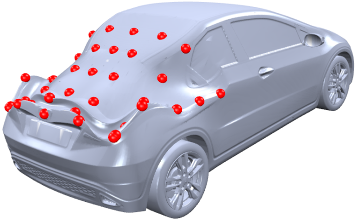

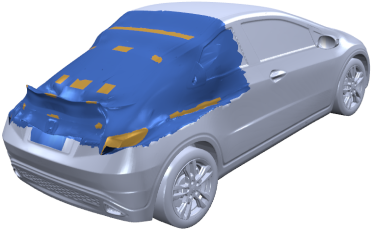

Conclusions & Future Work

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  - Strong coupling is important
- Future work:
  - Additional methods
  - Unified interface
  - Synthetic benchmarks

Thanks

... for your attention.

Questions?