Adaptive Space Deformations Based on Rigid Cells

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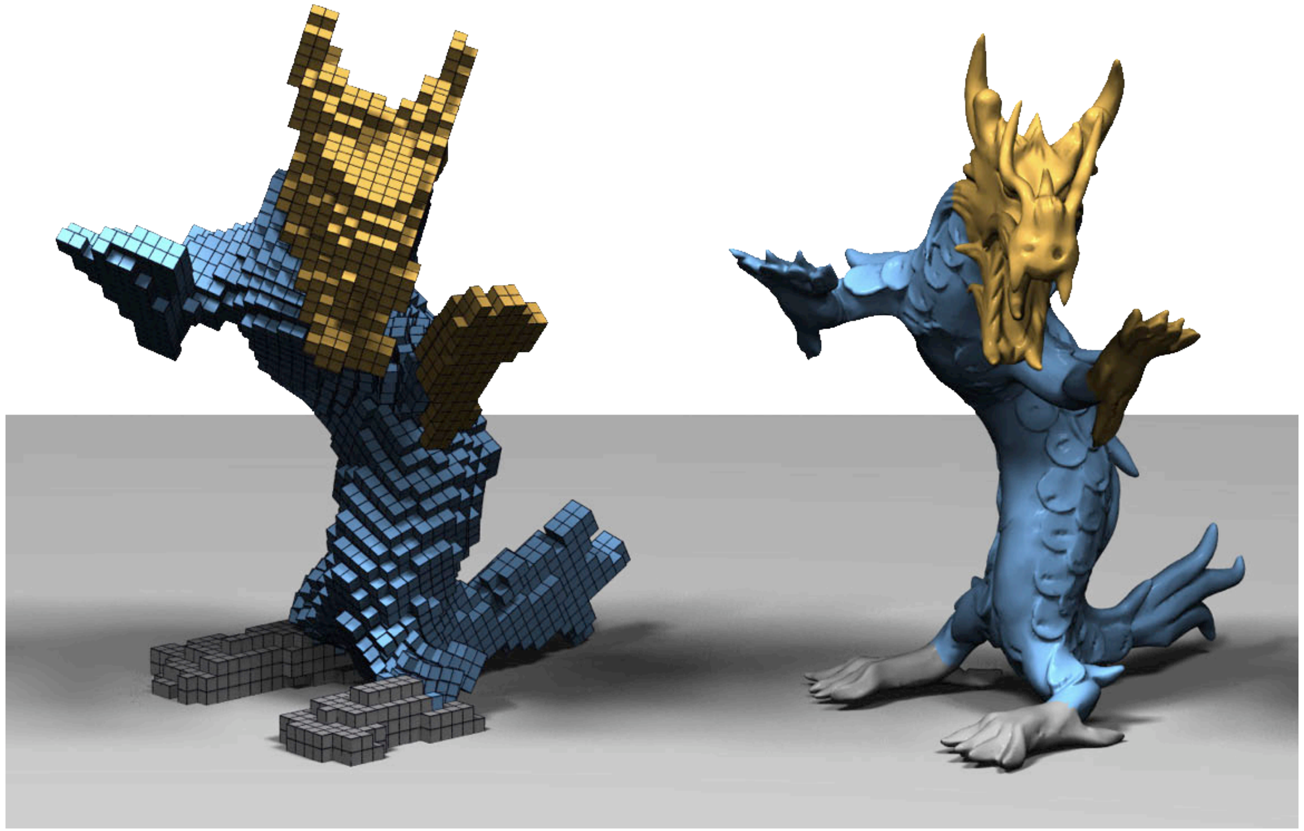

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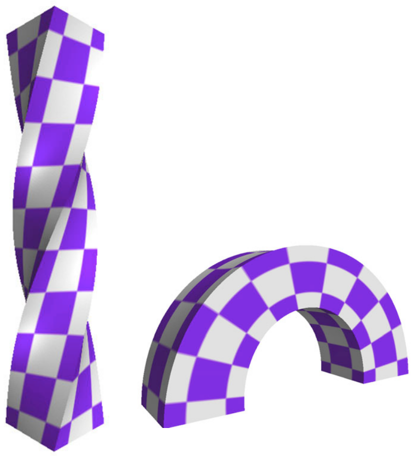
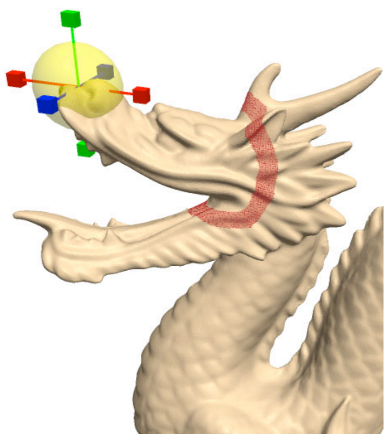
Linear Surface Deformation

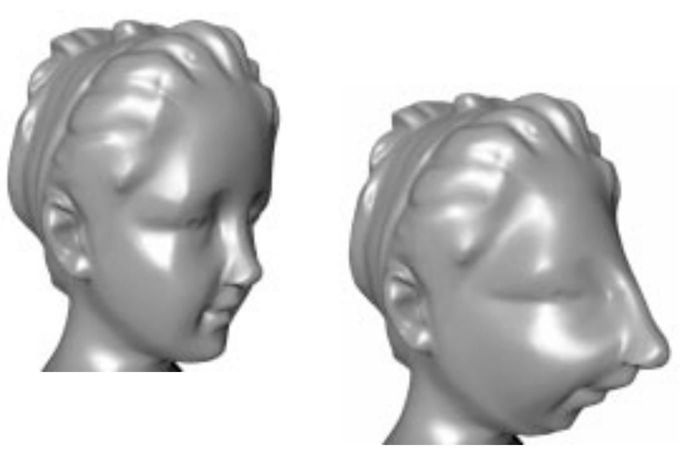
- Minimize quadratic surface energy
- Variational calculus
- Solve sparse linear system



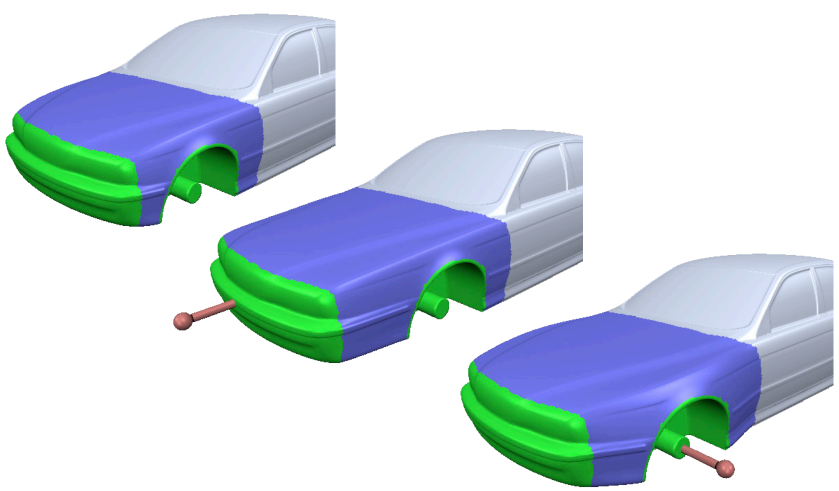
[Yu et al, SIG 04]



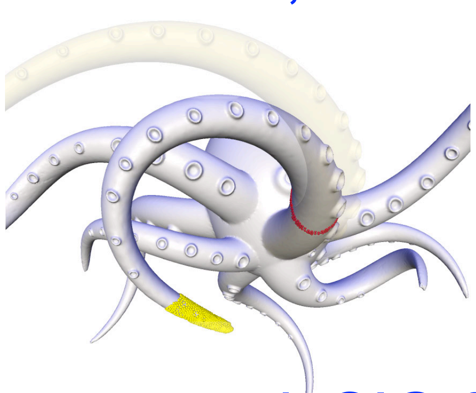
[Sorkine et al, SGP 04]



[Kobbelt et al, SIG 98]



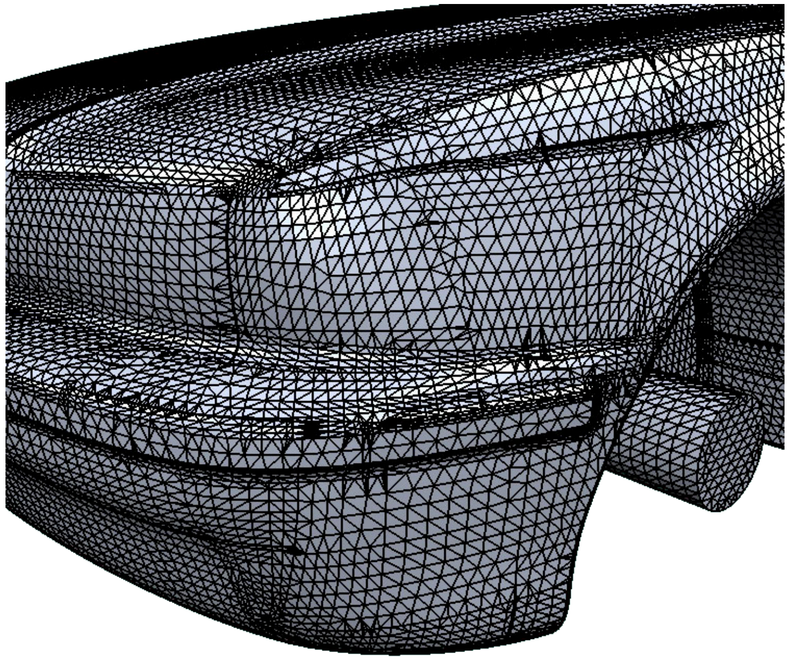
[Botsch & Kobbelt, SIG 04]



[Lipman et al, SIG 05]

Problematic Cases

Highly complex models

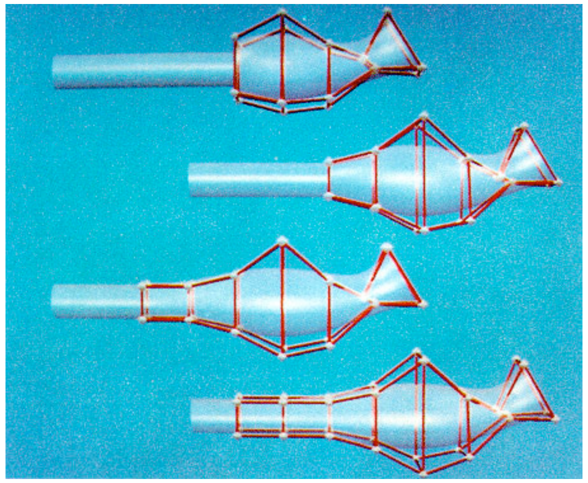


Geometric degeneracies

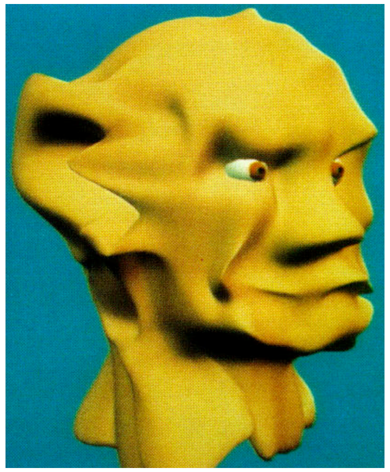
Topological inconsistencies

Linear Space Deformation

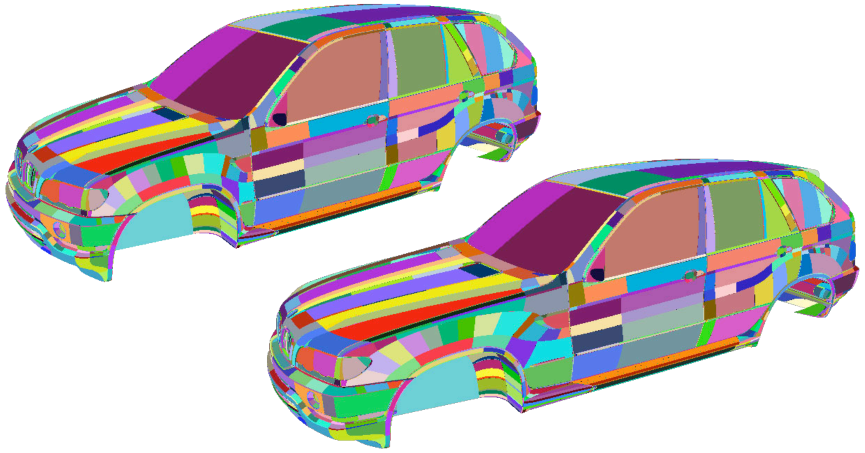
- Deform embedding space
- Again solve linear system, but...
- *Deformation complexity* << *surface complexity*



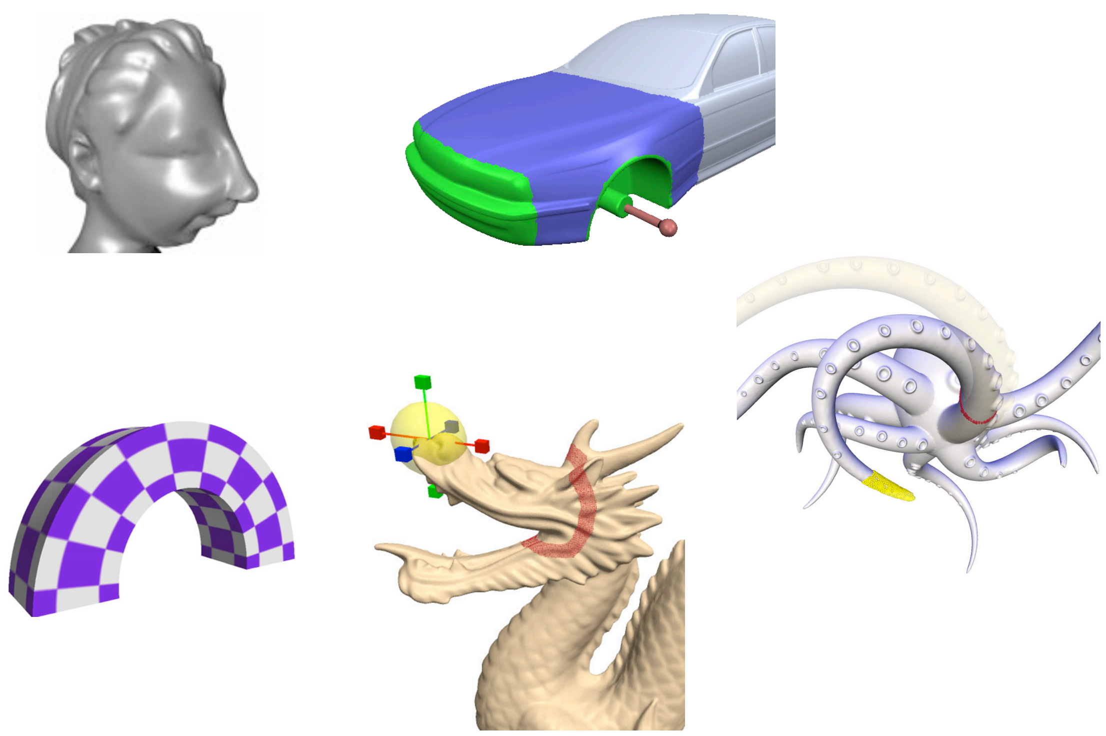
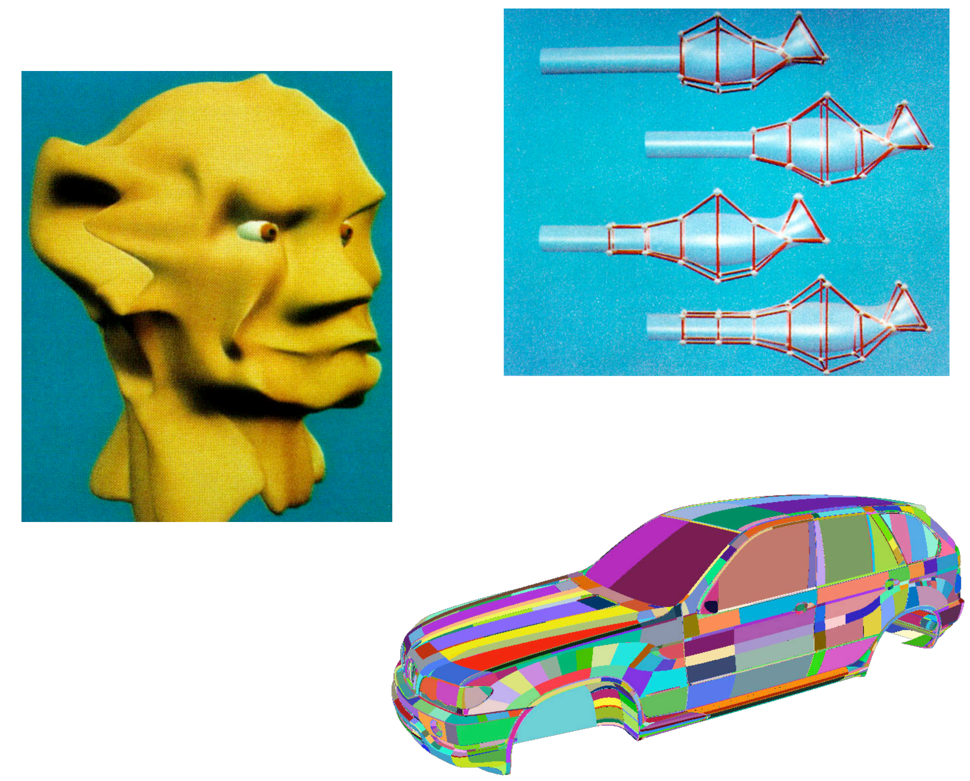
[Sederberg & Parry, SIG 86]



[Hsu et al, SIG 92]

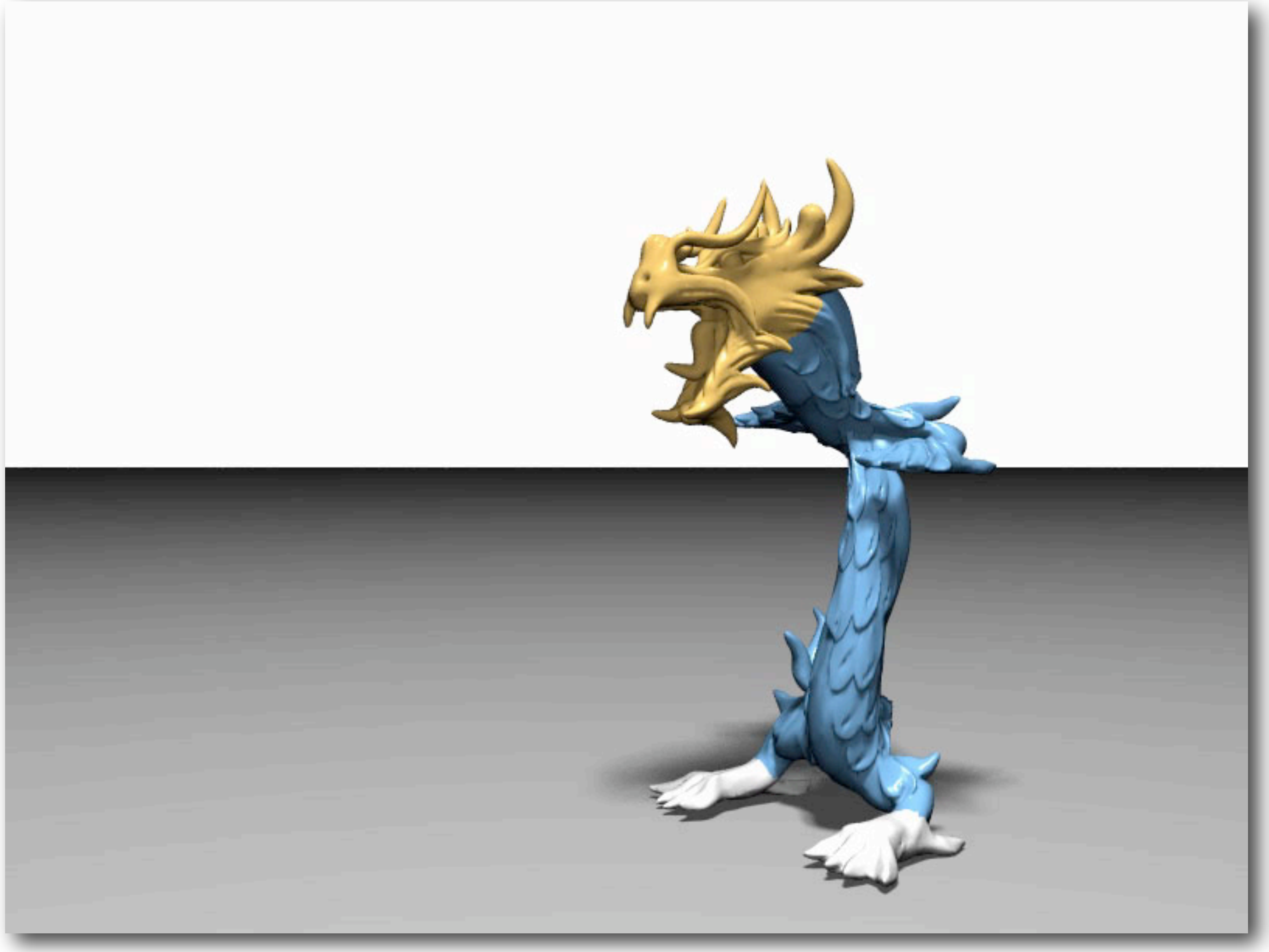


[Botsch & Kobbelt, EG 05]

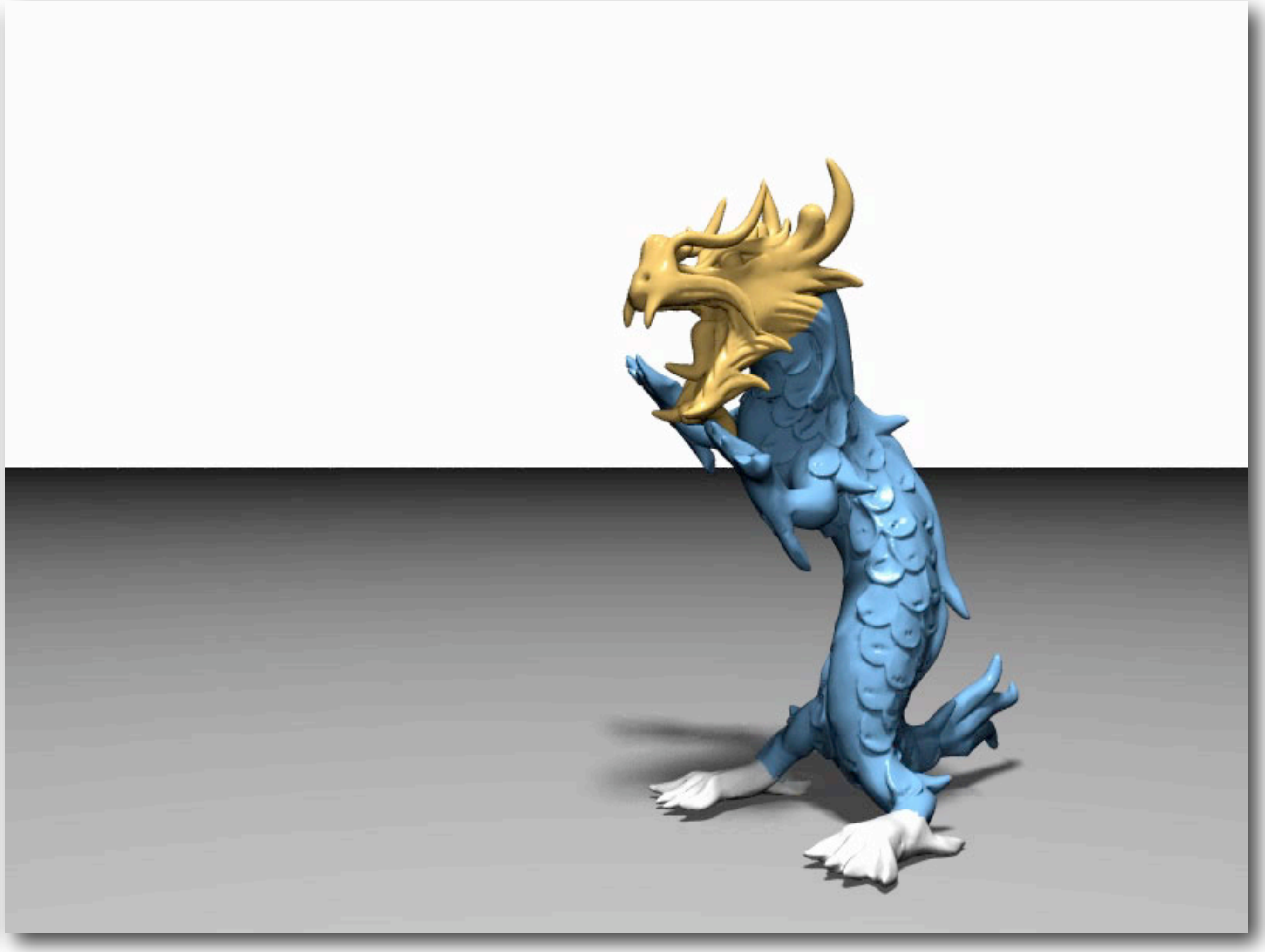
	Surface-Based	Space Deformation
Linear		
Nonlinear		

Linear vs. Nonlinear

- Problems with large deformations



Linear



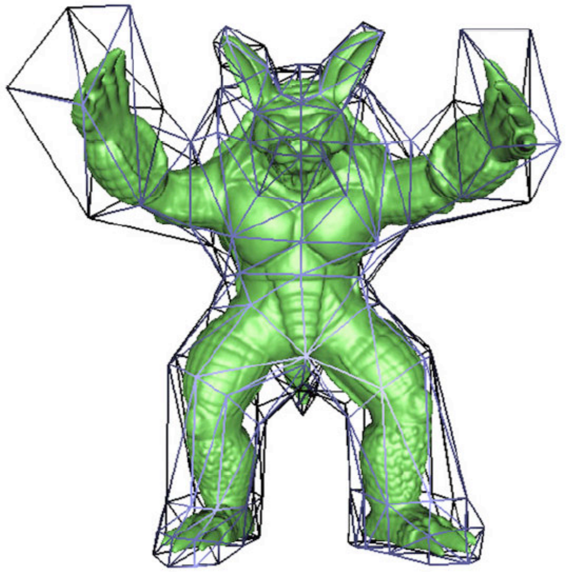
Nonlinear

Nonlinear Surface Deformation

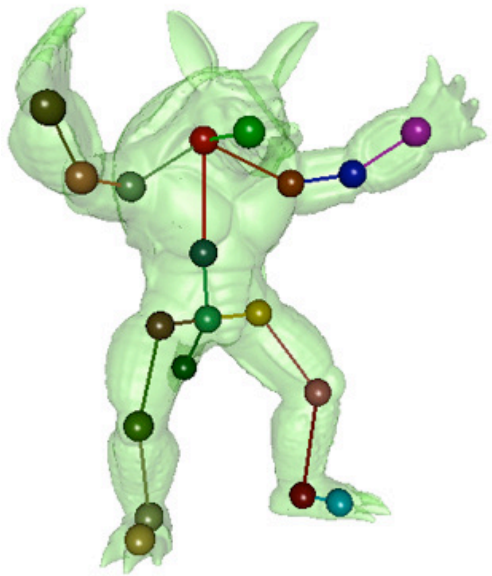
- Minimize nonlinear energies
  – Intuitive large-scale deformation
  – Robustness issues
  – Performance issues

[Au et al, SIG 07]

[Botsch et al, SGP 06]



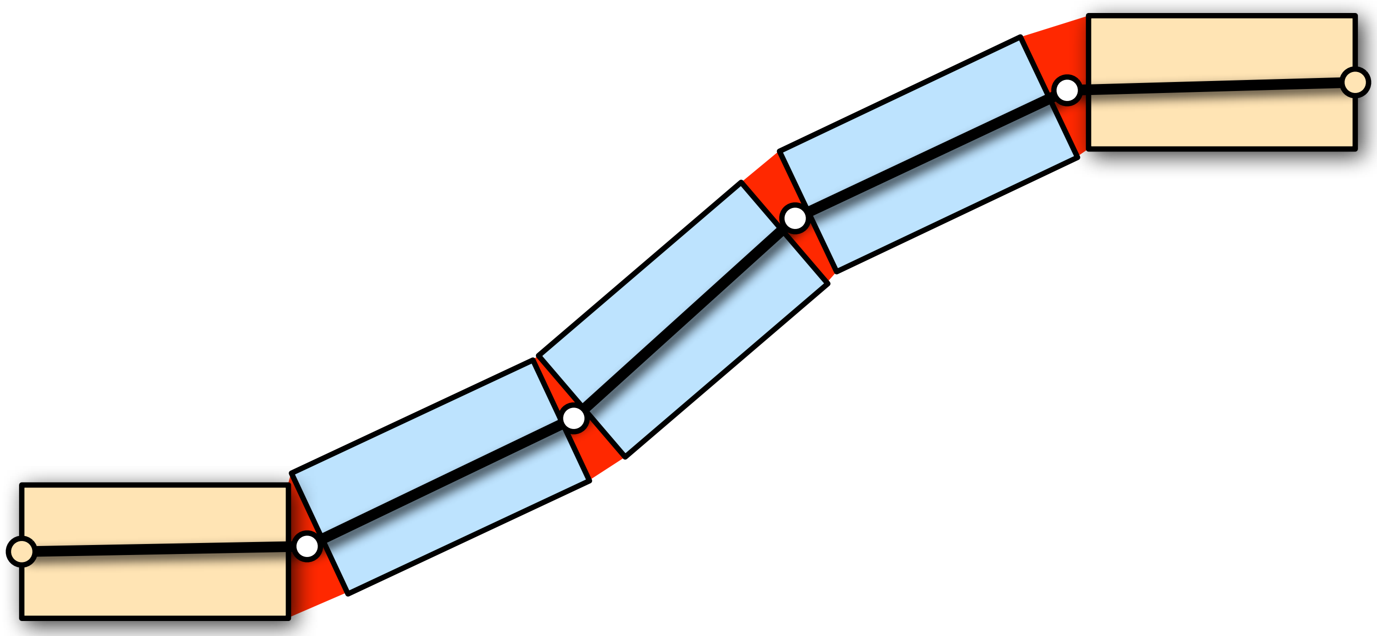
[Huang et al, SIG 06]



[Shi et al, SIG 07]

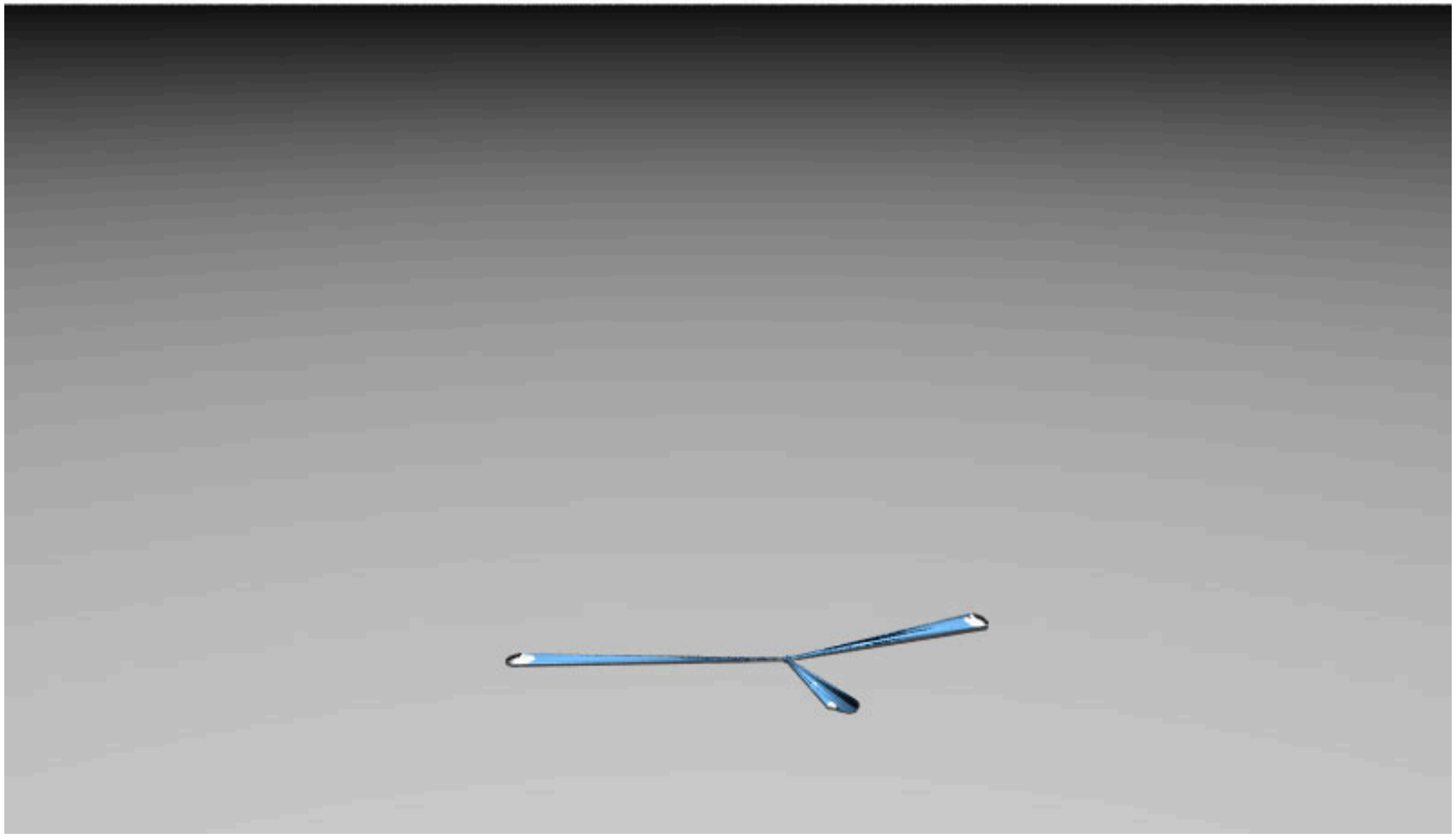
PriMo [Botsch et al, SGP 2006]

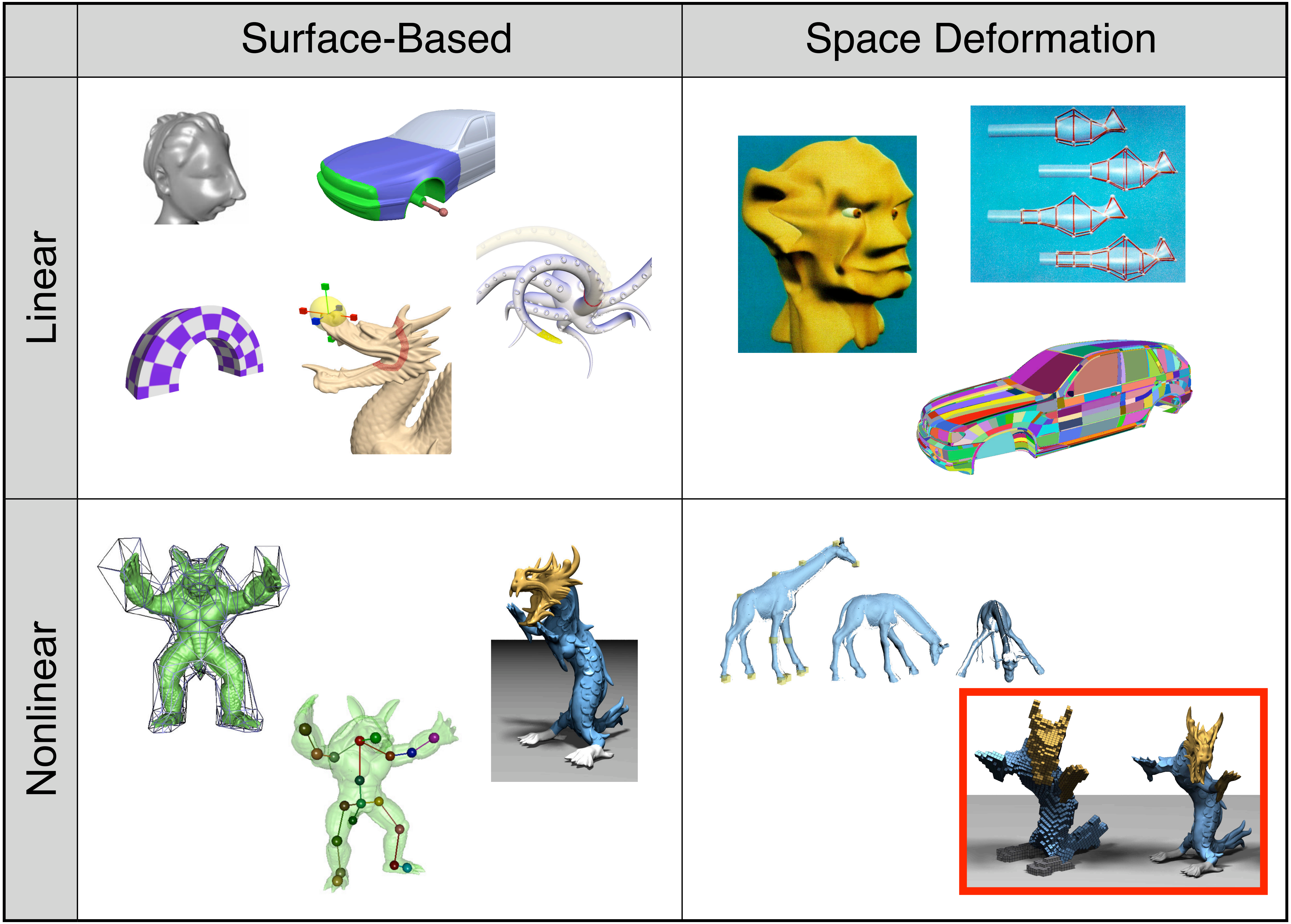
1. Extrude triangles to prisms / cells
2. Prescribes position/orientation for cells
3. Find optimal *rigid motions* per cell
4. Update vertices by averaged cell transformations



PriMo [Botsch et al, SGP 2006]

Rigidity → Robust optimization




Surface-Based
Space Deformation
Linear
Nonlinear

Design decisions:

Design decisions:

– Realistic behavior ⇒ Physically plausible

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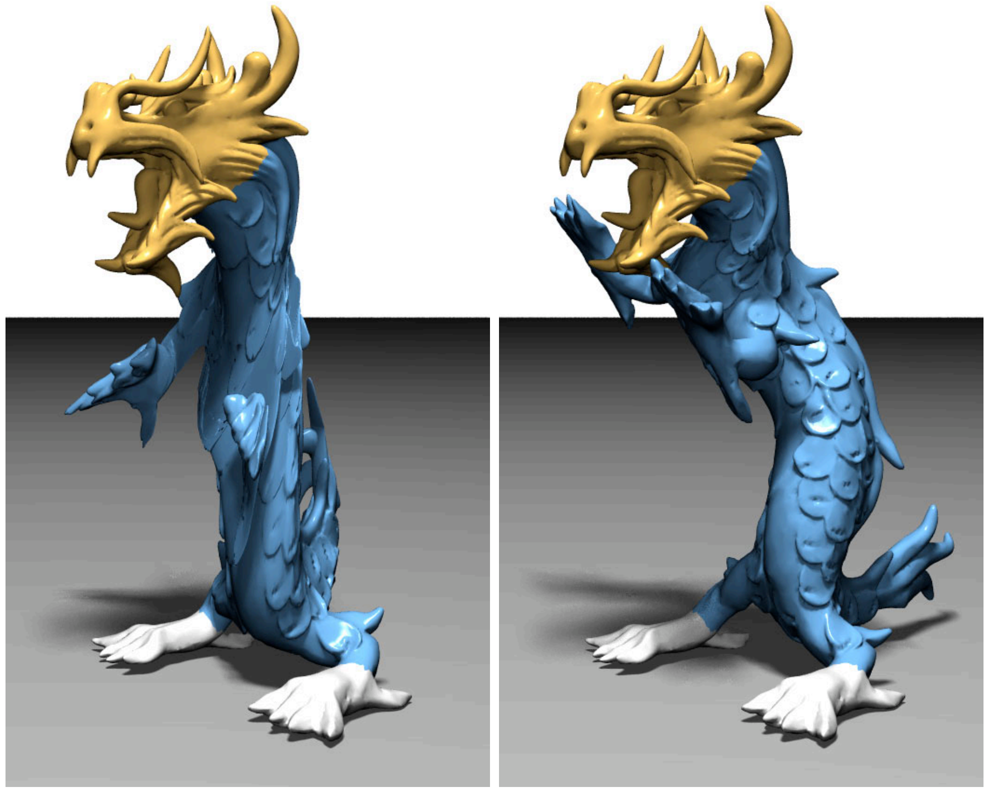
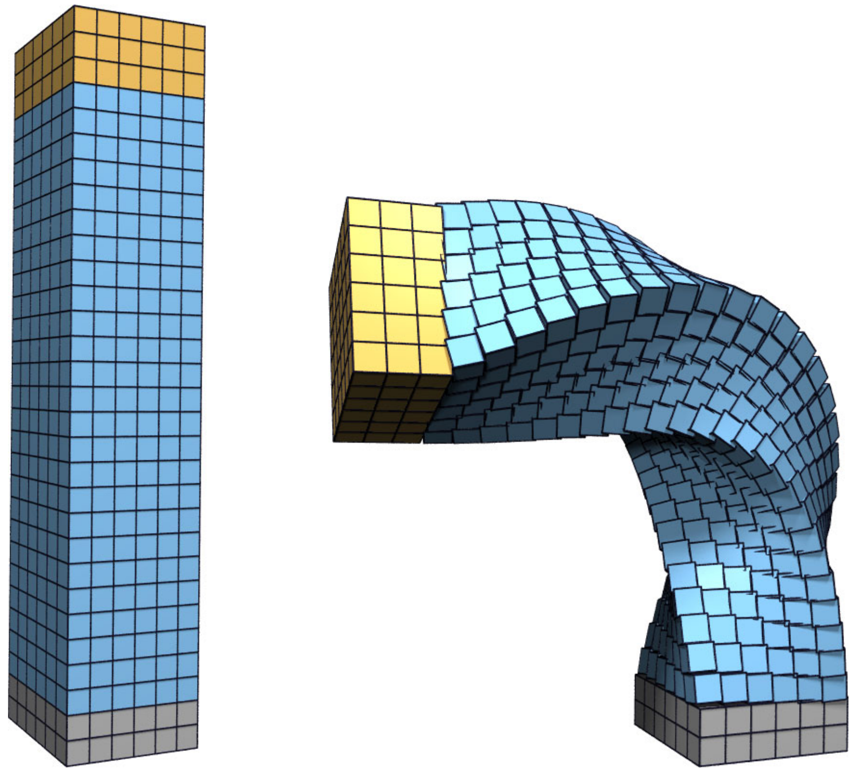
- Large deformations ⇒ Nonlinear energy



Design decisions:

- Realistic behavior ⇒ Physically plausible
- Large deformations ⇒ Nonlinear energy
- Robustness ⇒ Rigid cells



Design decisions:

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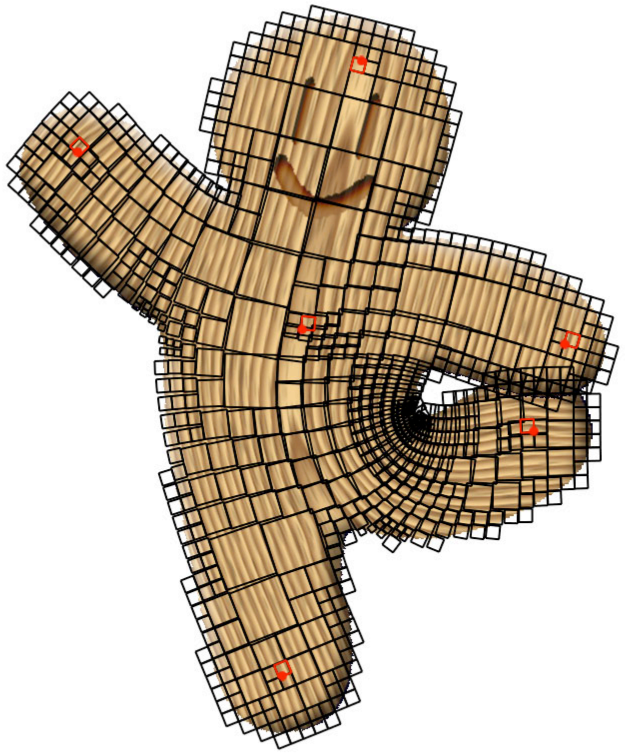
- Robustness ⇒ Rigid cells
- Applicability ⇒ Space deformation

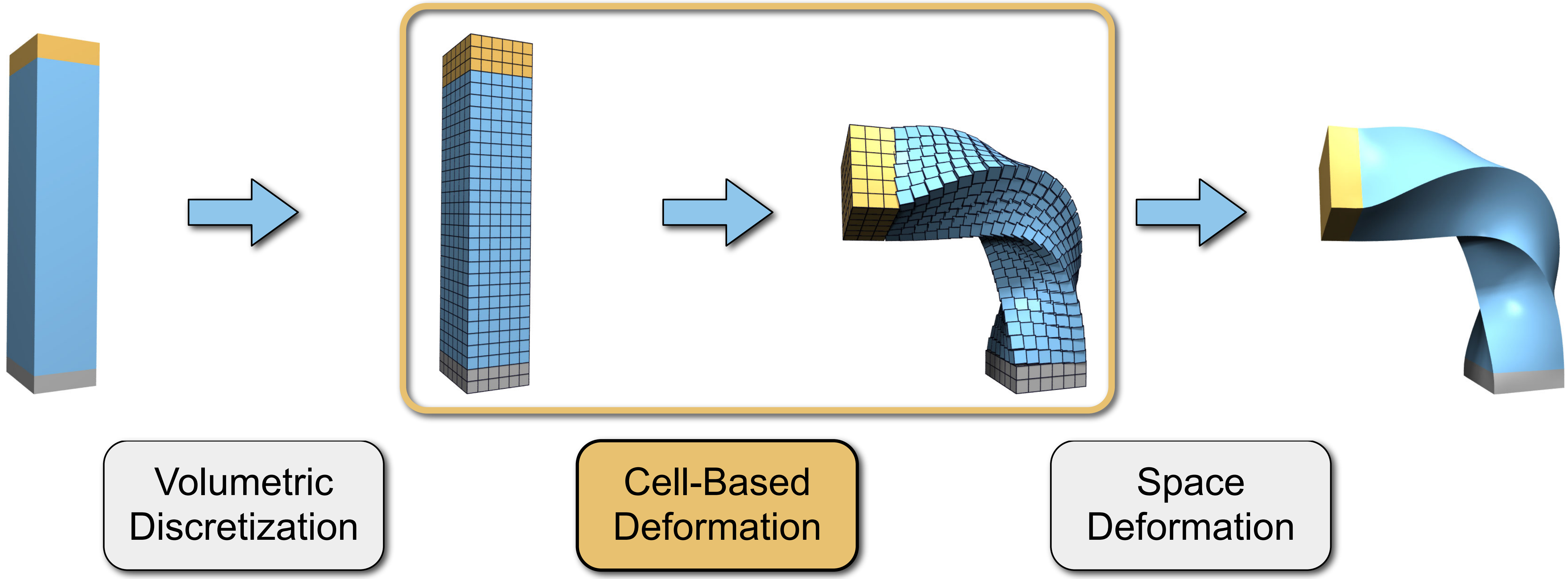
Adaptive Space Deformations Based on Rigid Cells

Design decisions:

- Realistic behavior ⇒ Physically plausible
- Large deformations ⇒ Nonlinear energy
- Robustness ⇒ Rigid cells
- Applicability ⇒ Space deformation
- Performance ⇒ Adaptive discretization



Deformation Pipeline



Deformation Energy

- Continuum mechanics
  - Strain energy defined by displacement's gradient
  - Local variation of displacement causes stretching
- Discrete rigid cells
  - Each cell stores rigid motion $\mathbf{T}_i(\mathbf{x}) = \mathbf{R}_i(\mathbf{x}) + \mathbf{t}_i$
  - Local differences of rigid transformations

Difference of Transformations

- Frobenius norm of matrices
  - Geometric meaning of matrix elements?

$$\|\mathbf{T}_i - \mathbf{T}_j\|^2 \;:=\; \sum_{k=1}^{4}\sum_{l=1}^{4}\Big((\mathbf{T}_i)_{k,l} - (\mathbf{T}_j)_{k,l}\Big)^2$$

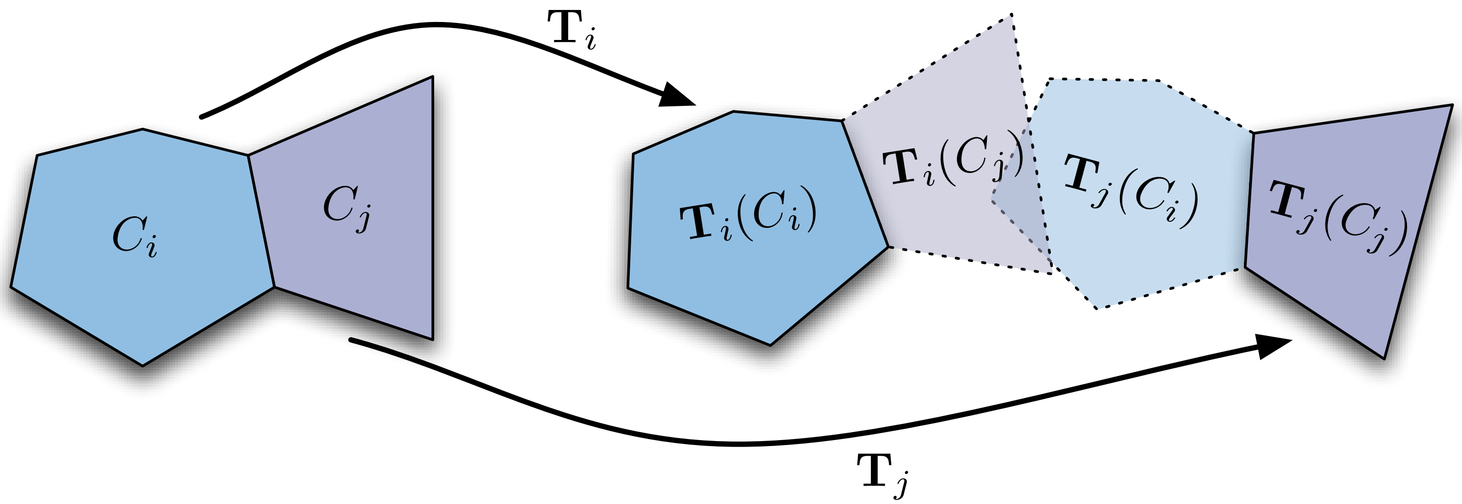
- [Pottmann et al, 04]
  - Difference of images of sample points (which?)

$$\|\mathbf{T}_i - \mathbf{T}_j\|^2 \;:=\; \frac{1}{k}\sum_{l=1}^{k}\|\mathbf{T}_i(\mathbf{x}_l) - \mathbf{T}_j(\mathbf{x}_l)\|^2$$

Nonlinear Energy

- Integrate over neighboring cells' interiors

$$E_{ij}(\mathbf{T}_i, \mathbf{T}_j) \;:=\; \int_{C_i \cup C_j} \left\| \mathbf{T}_i(\mathbf{x}) - \mathbf{T}_j(\mathbf{x}) \right\|^2 \mathrm{d}\mathbf{x}$$



Nonlinear Energy

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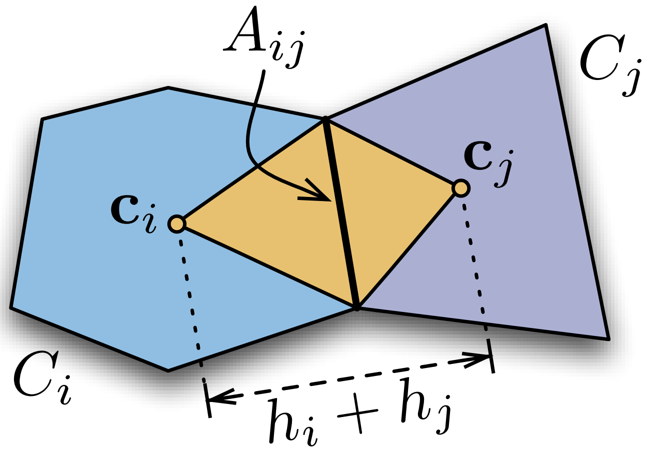
$$E_{ij}(\mathbf{T}_i, \mathbf{T}_j) \;:=\; \int_{C_i \cup C_j} \left\| \mathbf{T}_i(\mathbf{x}) - \mathbf{T}_j(\mathbf{x}) \right\|^2 \mathrm{d}\mathbf{x}$$

- Accumulated global energy

$$E(\mathbf{T}_1, \ldots, \mathbf{T}_n) \;=\; \sum_{\{i,j\}} w_{ij} \cdot E_{ij}(\mathbf{T}_i, \mathbf{T}_j)$$

$$w_{ij} \;=\; w_{ji} \;=\; \frac{A_{ij}}{h_i + h_j}$$



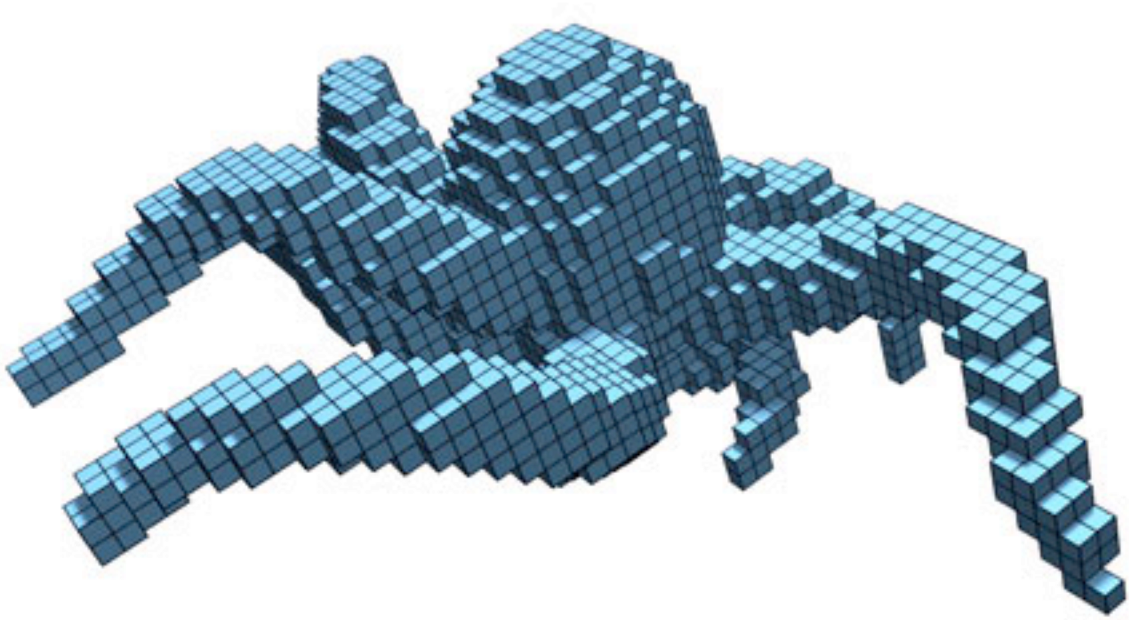
Nonlinear Minimization

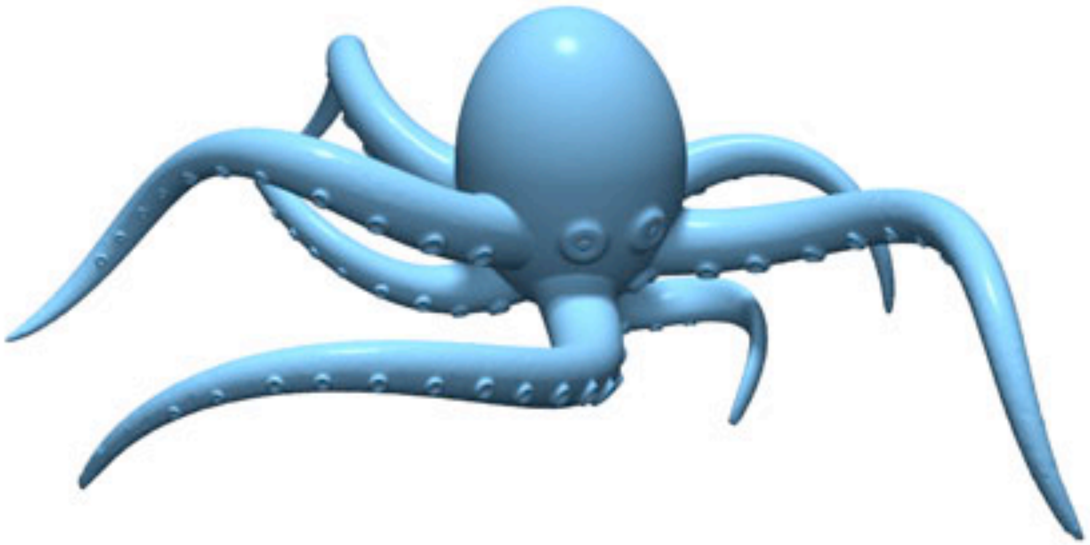
- Find *rigid* motion $\mathbf{T}_i$ per cell C_i

$$\min_{\{\mathbf{T}_i\}} \sum_{\{i,j\}} w_{ij} \int_{C_i \cup C_j} \left\| \mathbf{T}_i(\mathbf{x}) - \mathbf{T}_j(\mathbf{x}) \right\|^2 \mathrm{d}\mathbf{x}$$

- Generalized shape matching [Botsch et al, SGP 06]
  - Robust geometric optimization
  - Nonlinear Newton-type minimization
  - See paper for details...

Robust Optimization





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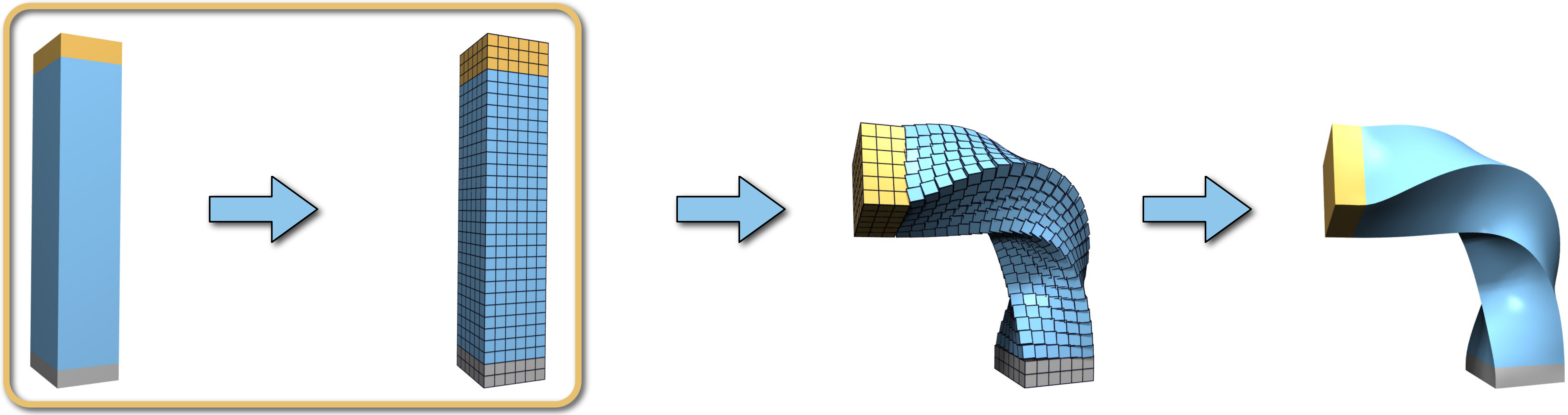


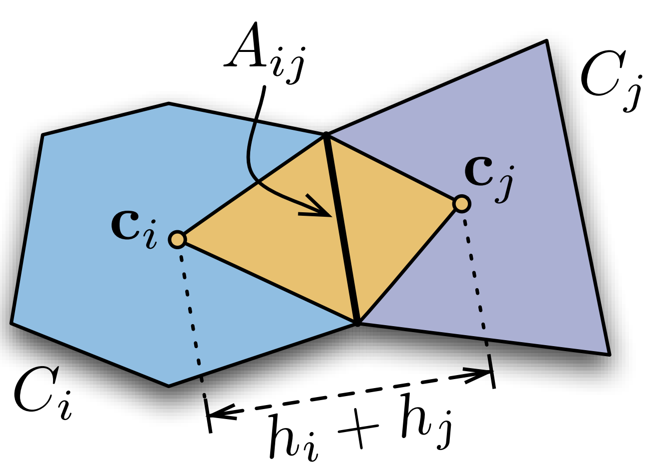
Volumetric Discretization

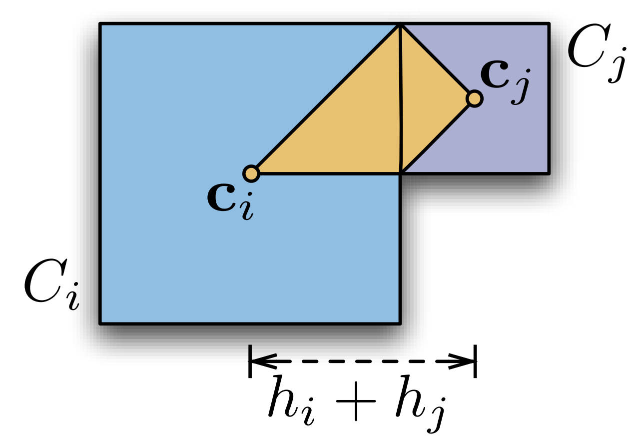
Cell-Based Deformation

Space Deformation

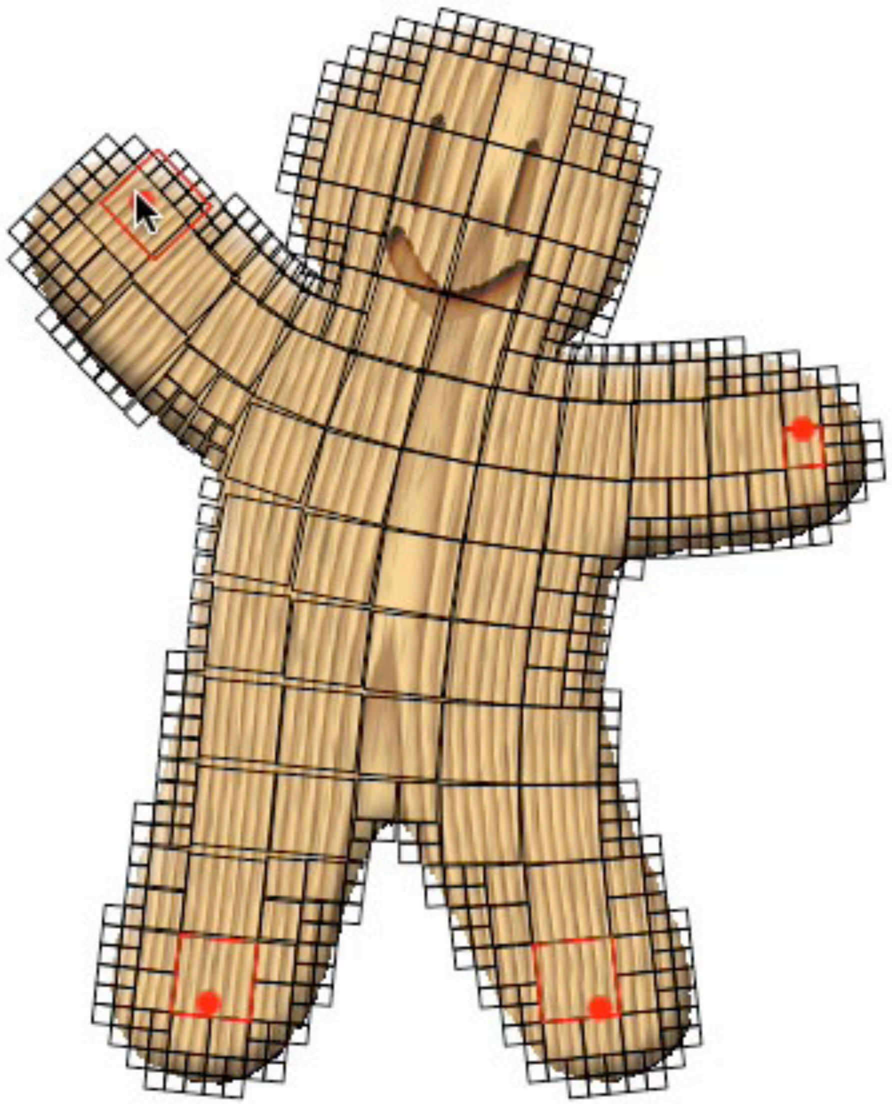
Volumetric Discretization

- Octree-like adaptive voxelization
  - Easy to implement, efficient to compute
  - Analytic integration
- T-junctions are *no problem* !

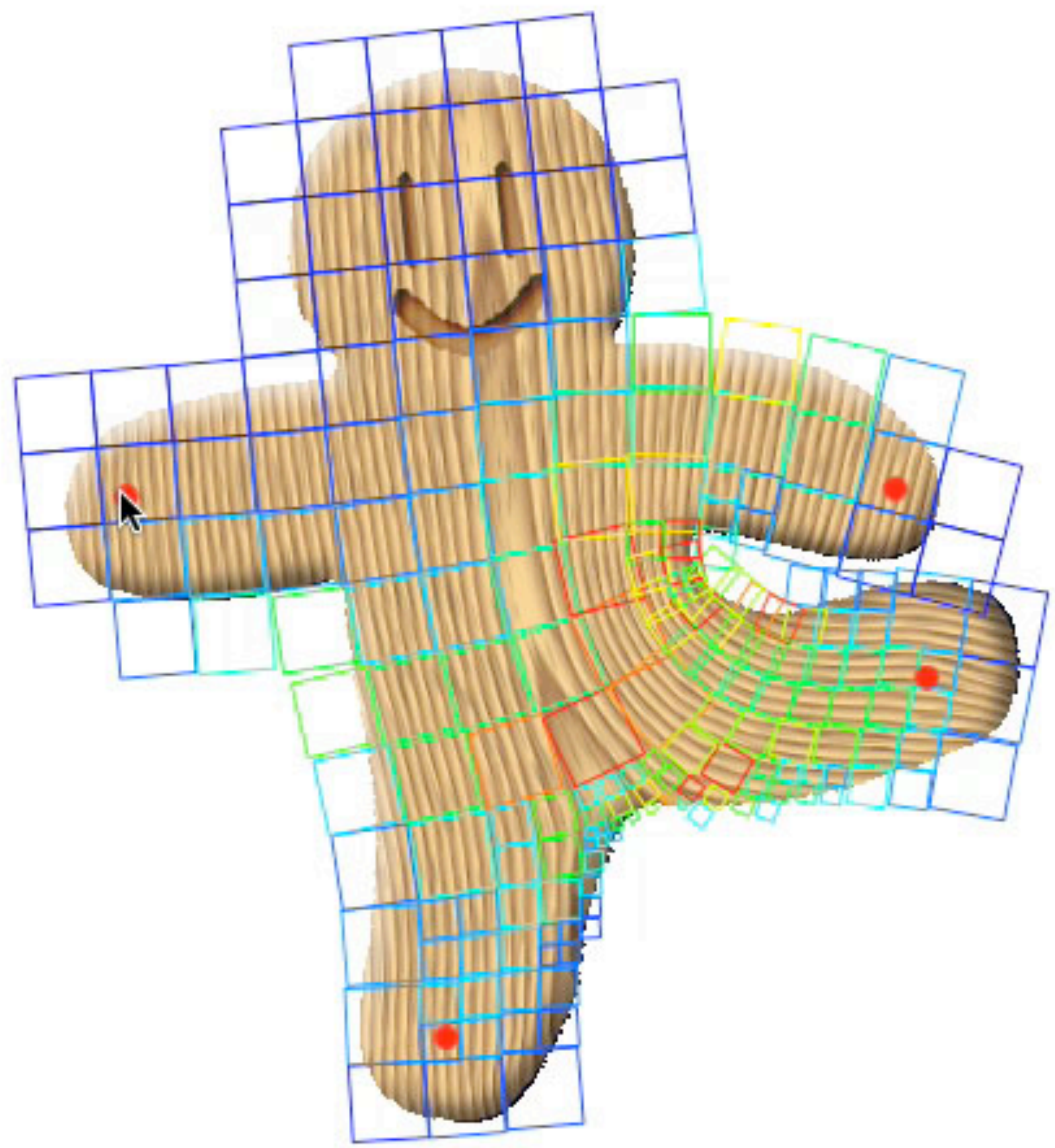




Boundary Refinement



Energy-Driven Refinement



3D Adaptive Refinement

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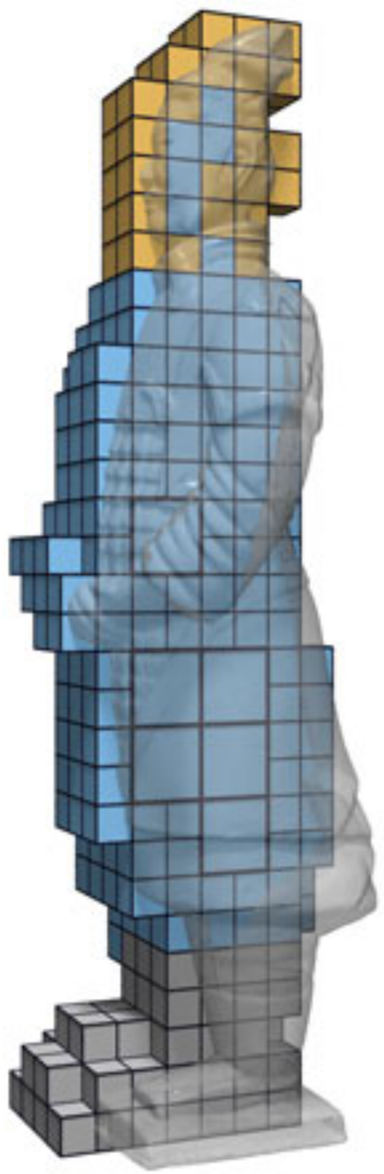

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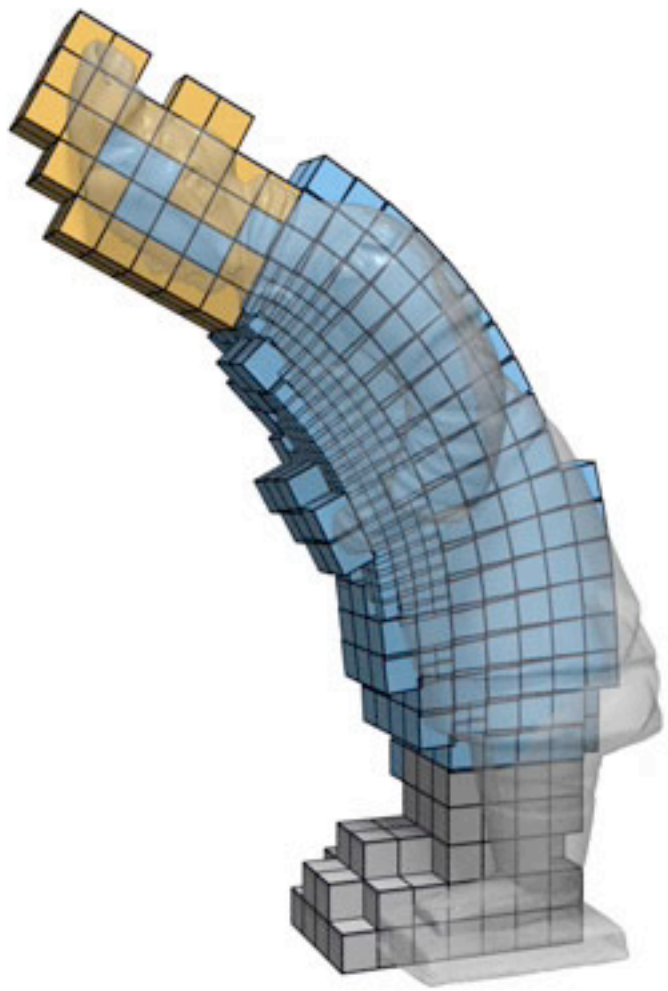
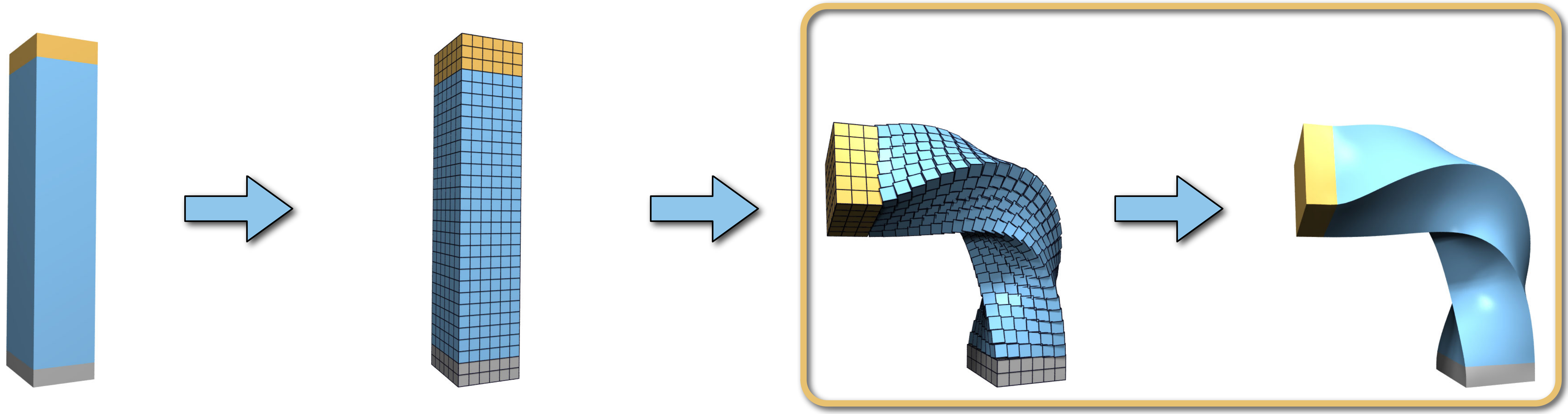


Deformation Pipeline

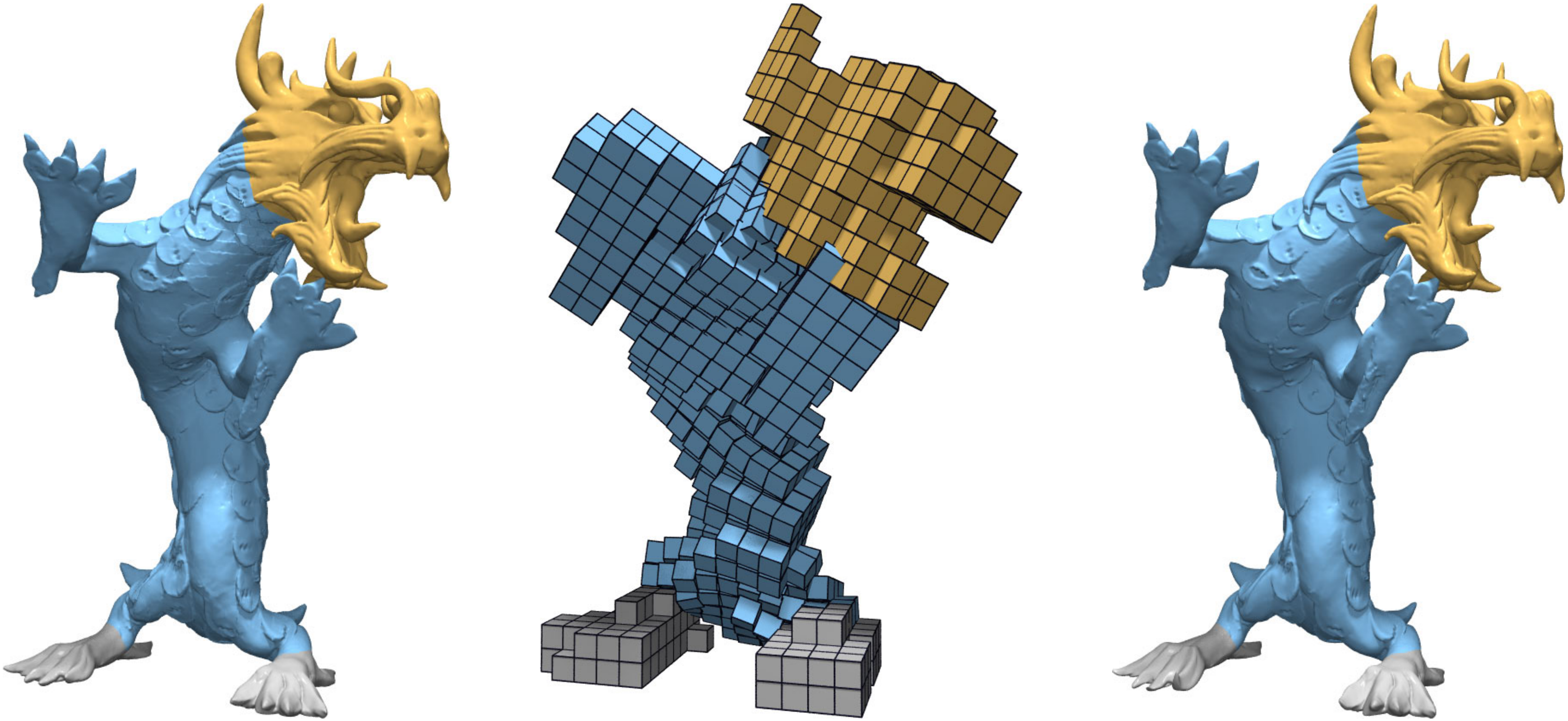


Volumetric Discretization

Cell-Based Deformation

Space Deformation

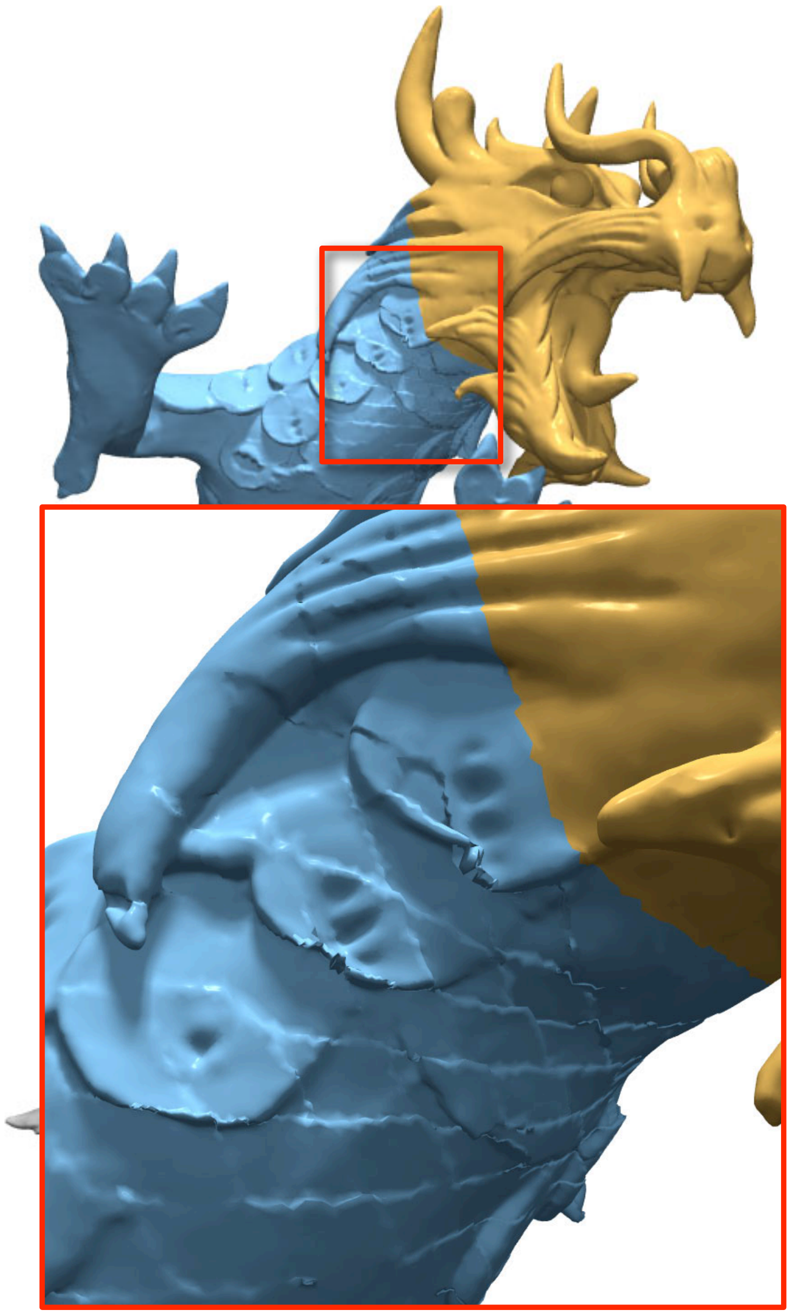
Deformation Interpolation



Preview:
Averaging / Skinning

Final:
RBF Interpolation

Deformation Interpolation



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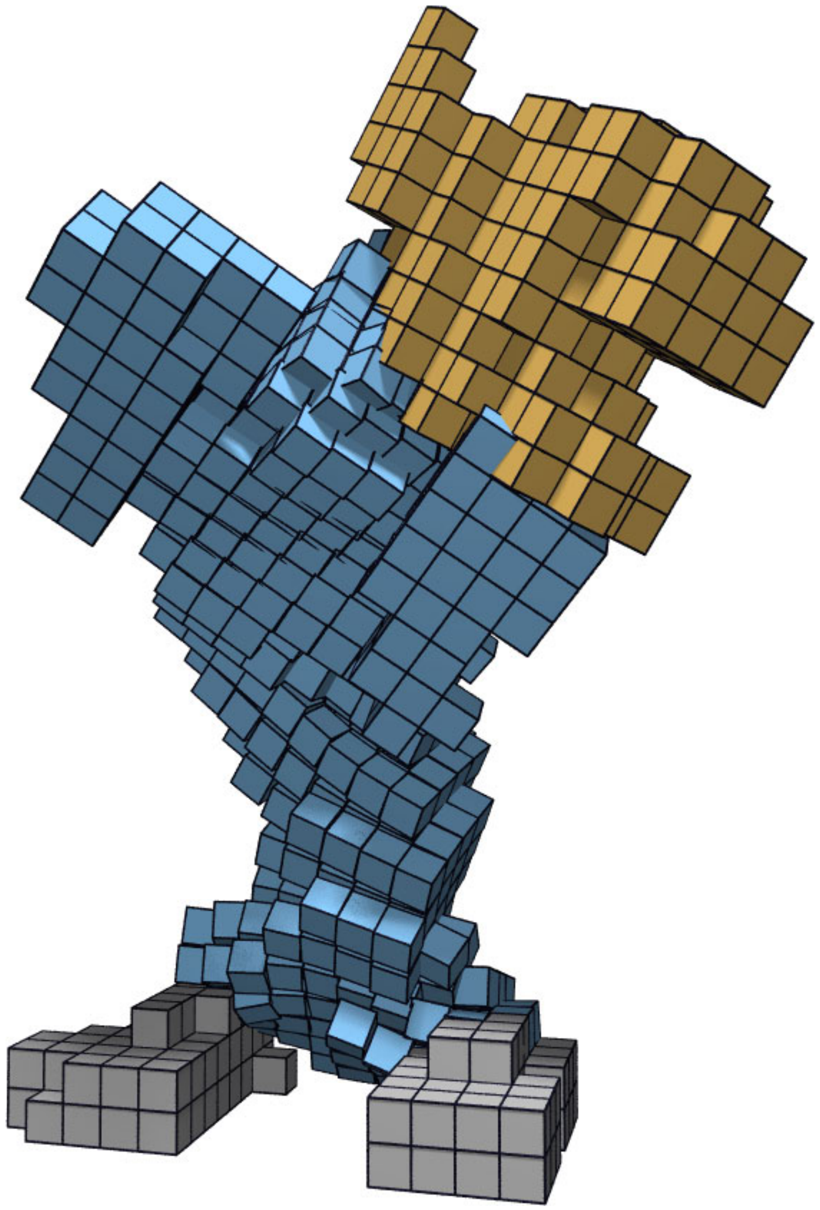

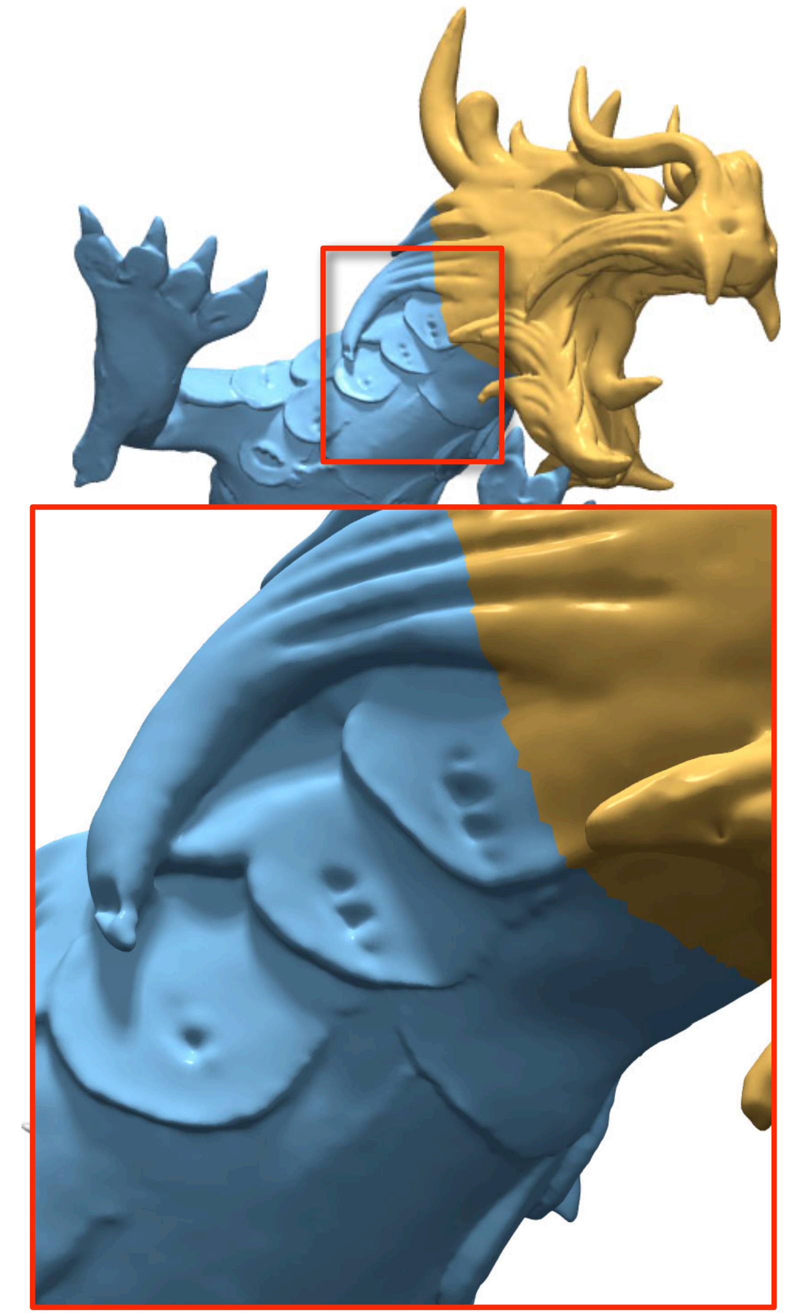

Final:
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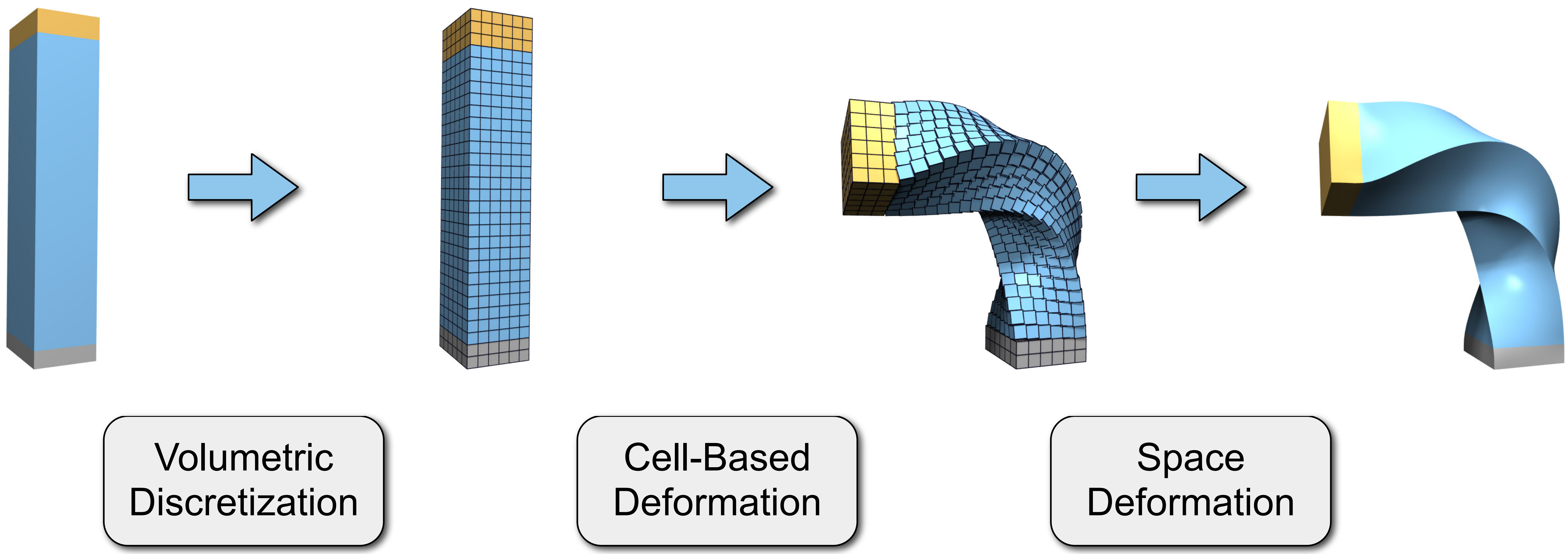
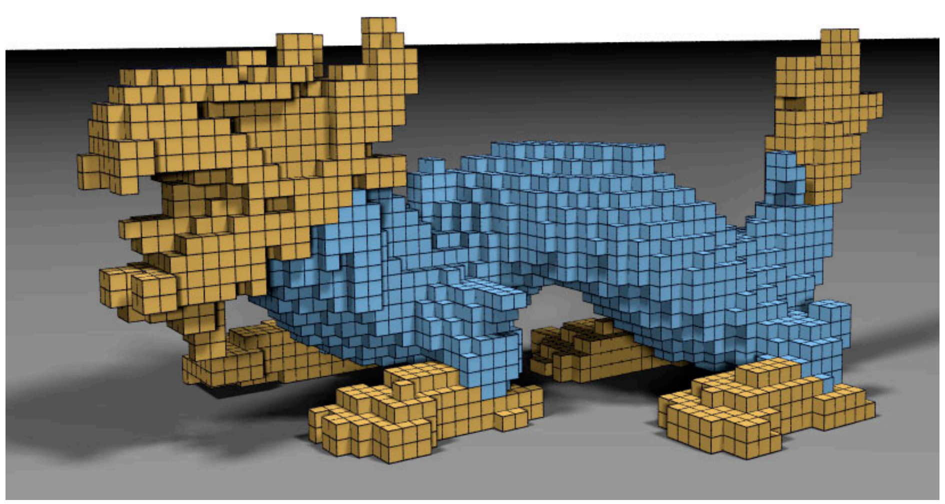


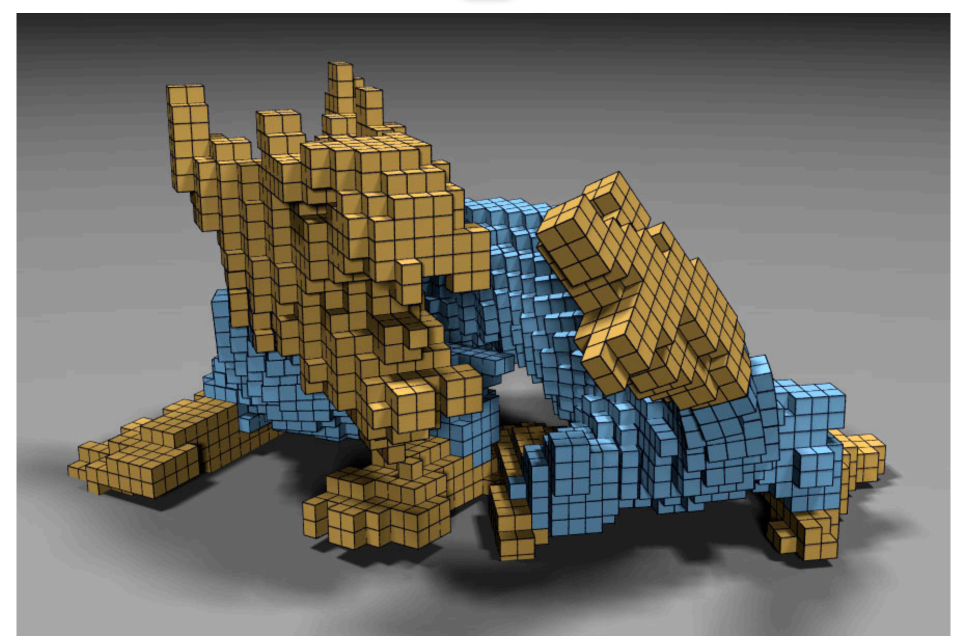
3D Deformation

Complex Models

100k



5.5k



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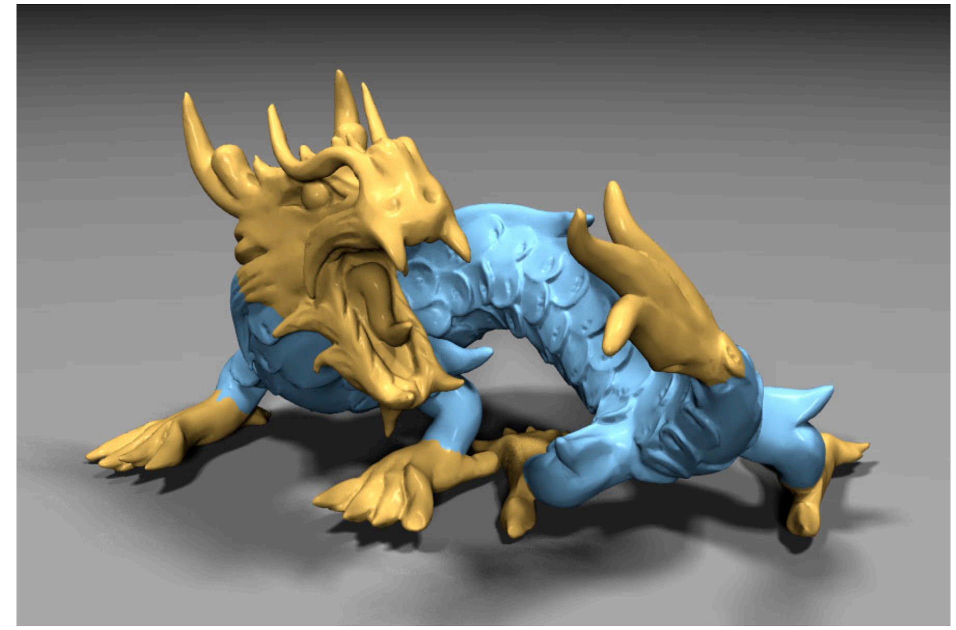

100k

Complex Models

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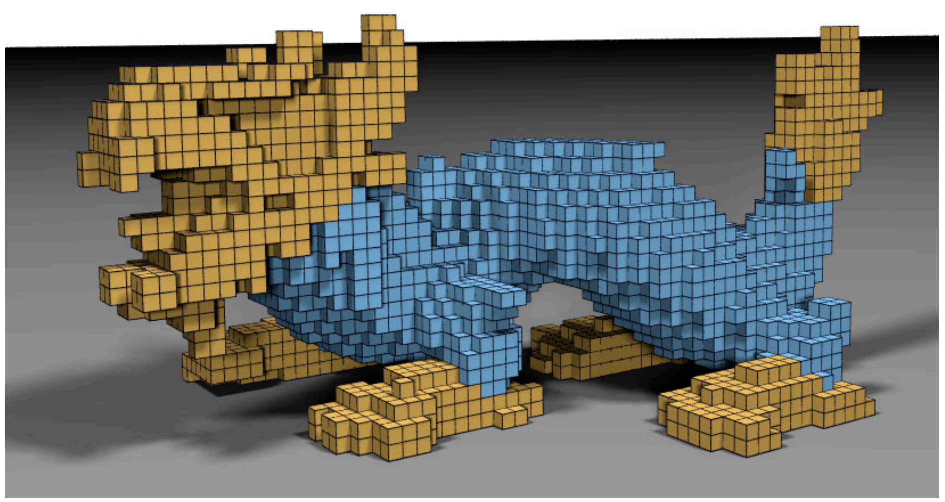

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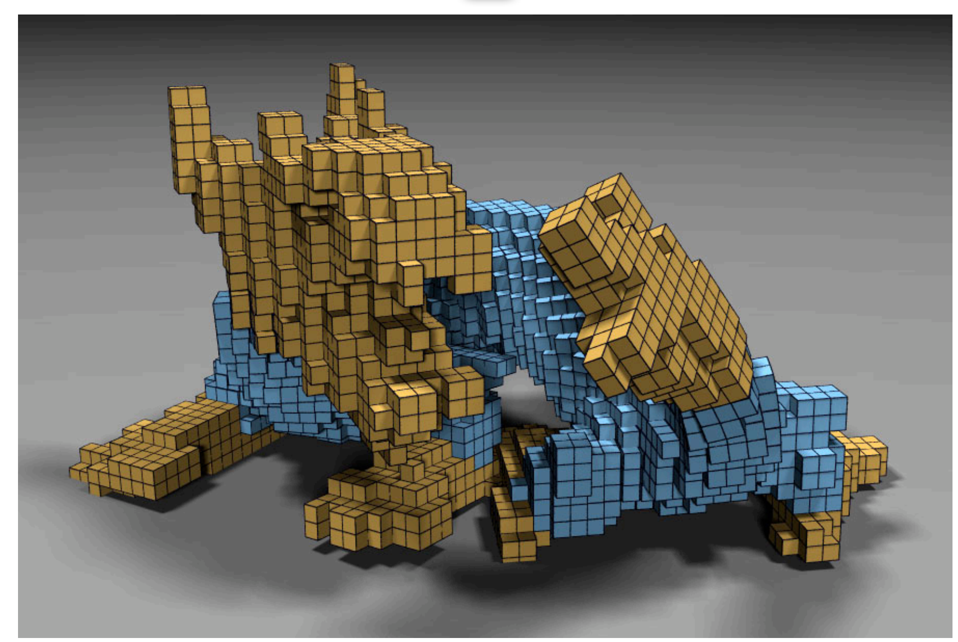

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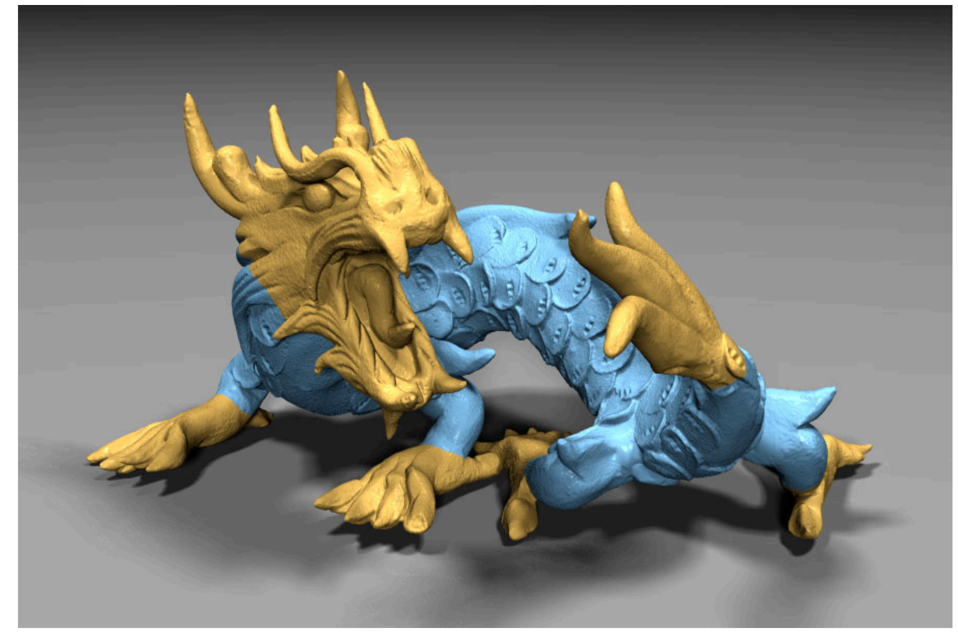
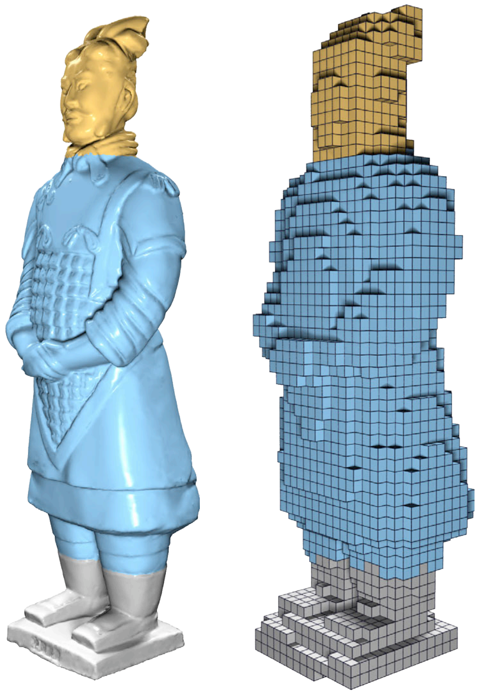
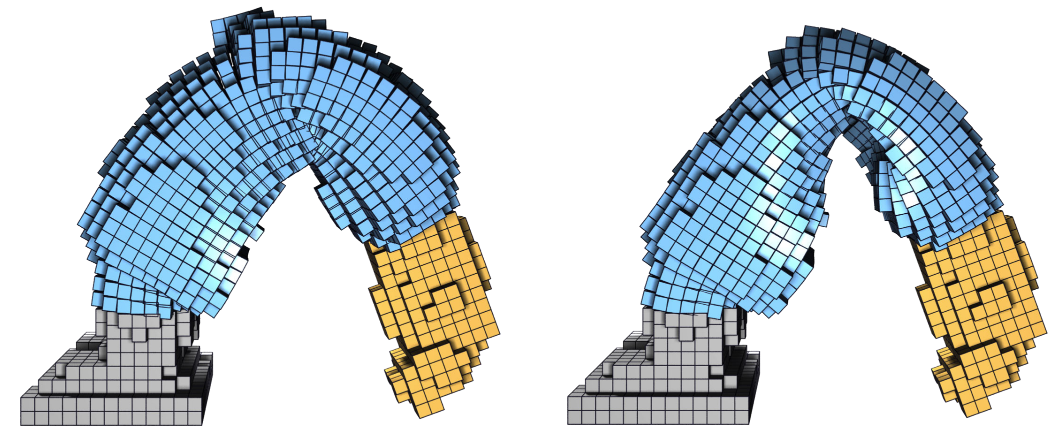
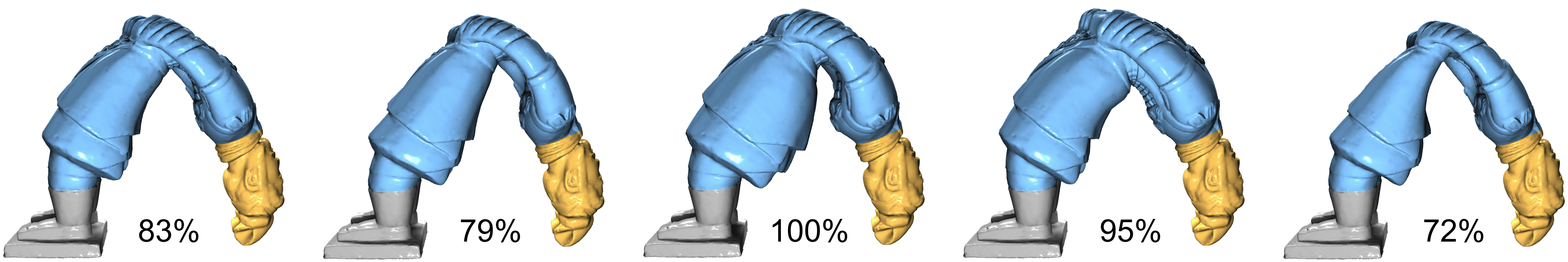


7M

Shell vs. Solid







Primo

discrete shell

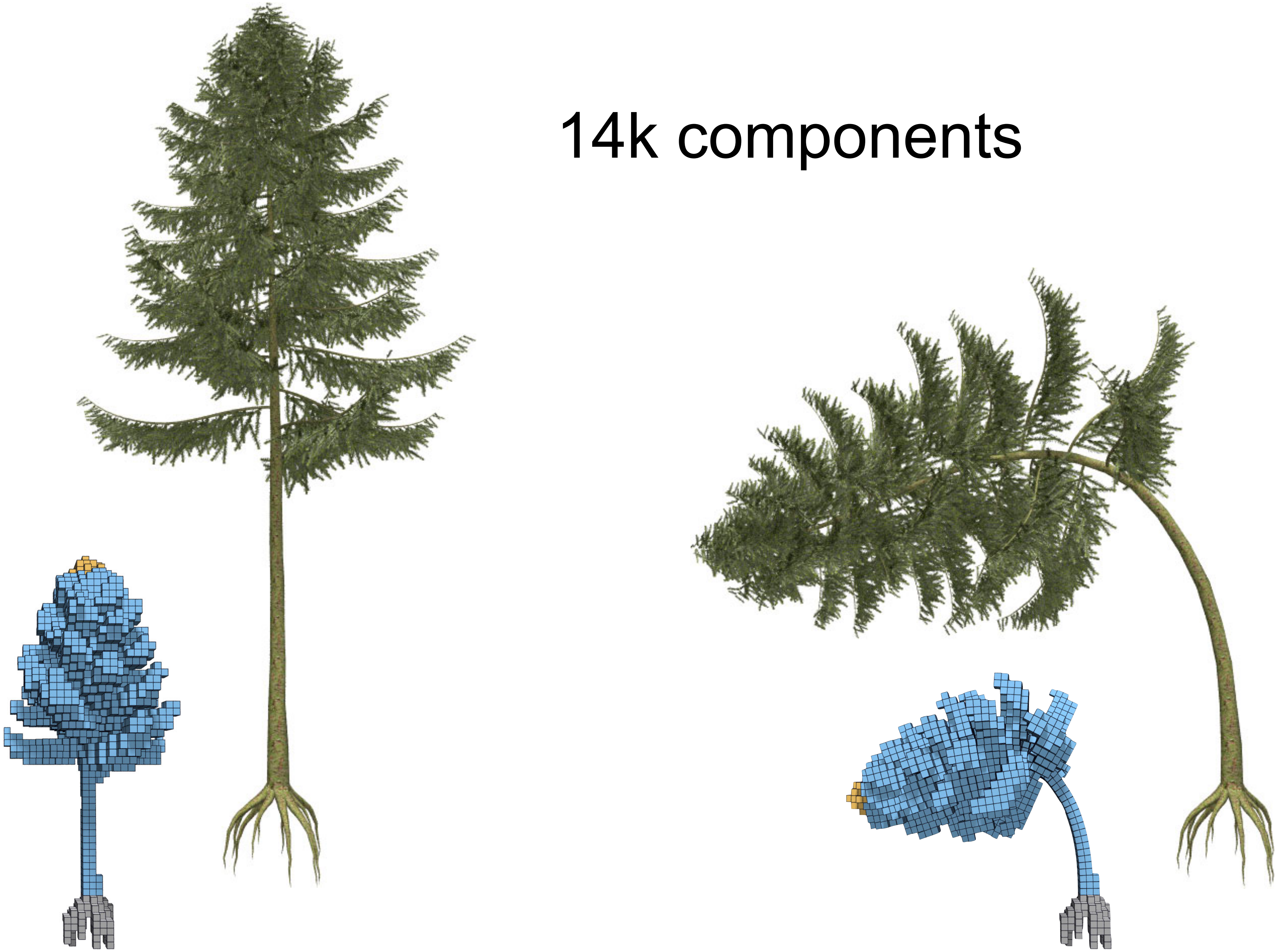
discrete shell
+ volume constr.

our approach
with interior cells

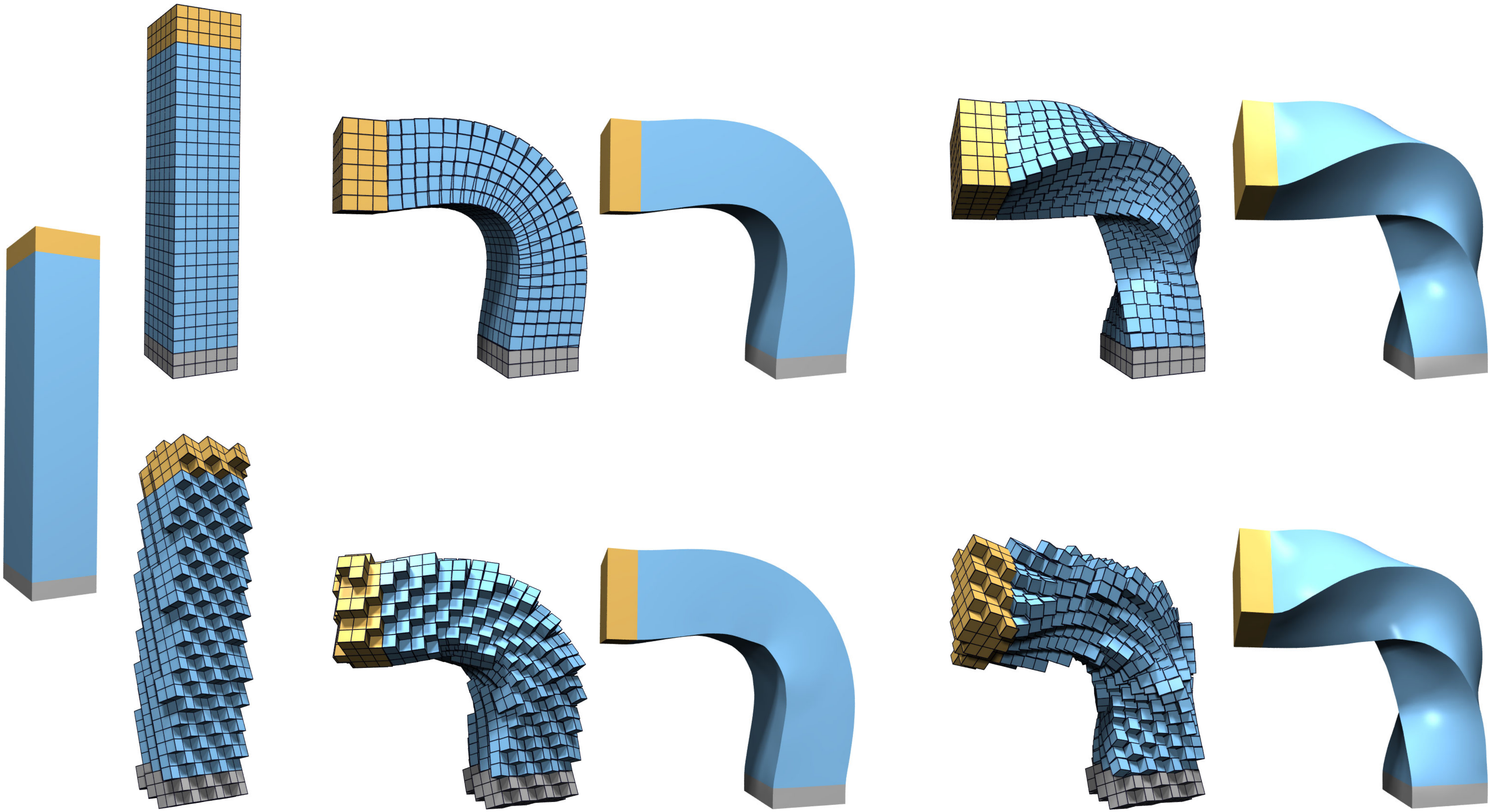
our approach
w/o interior cells

Triangle Soup

14k components



Aliasing



Conclusion

- Physically plausible deformation energy
  - Shells & solids, 2D & 3D
  - Arbitrary convex cell arrangements
  - Robust geometric shape matching
- Adaptive discretization
  - Static and dynamic refinement
- Smooth space deformation
  - Meshes, triangle soups, point sets, ...