

# Deformation Transfer for Detail-Preserving Surface Editing

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# Outline

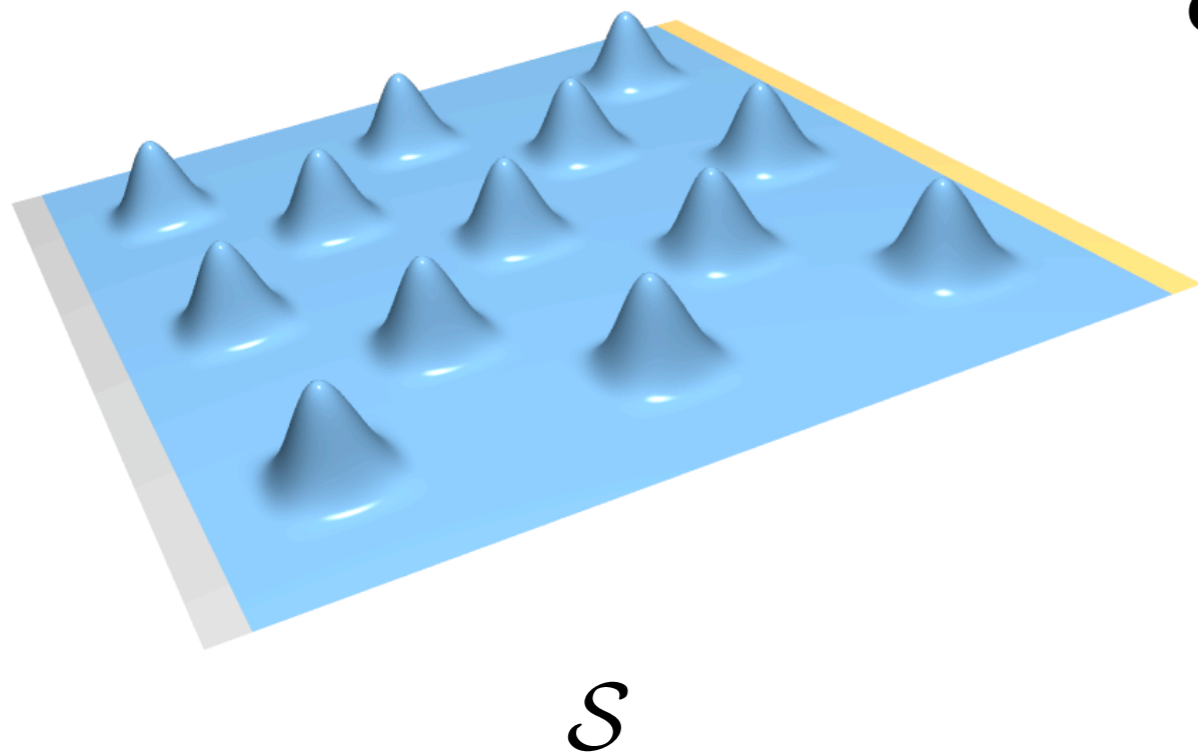
- Boundary constraint modeling
  - Gradient-based editing
  - Deformation transfer
- 
- Equivalence
  - Multiresolution Editing

# Outline

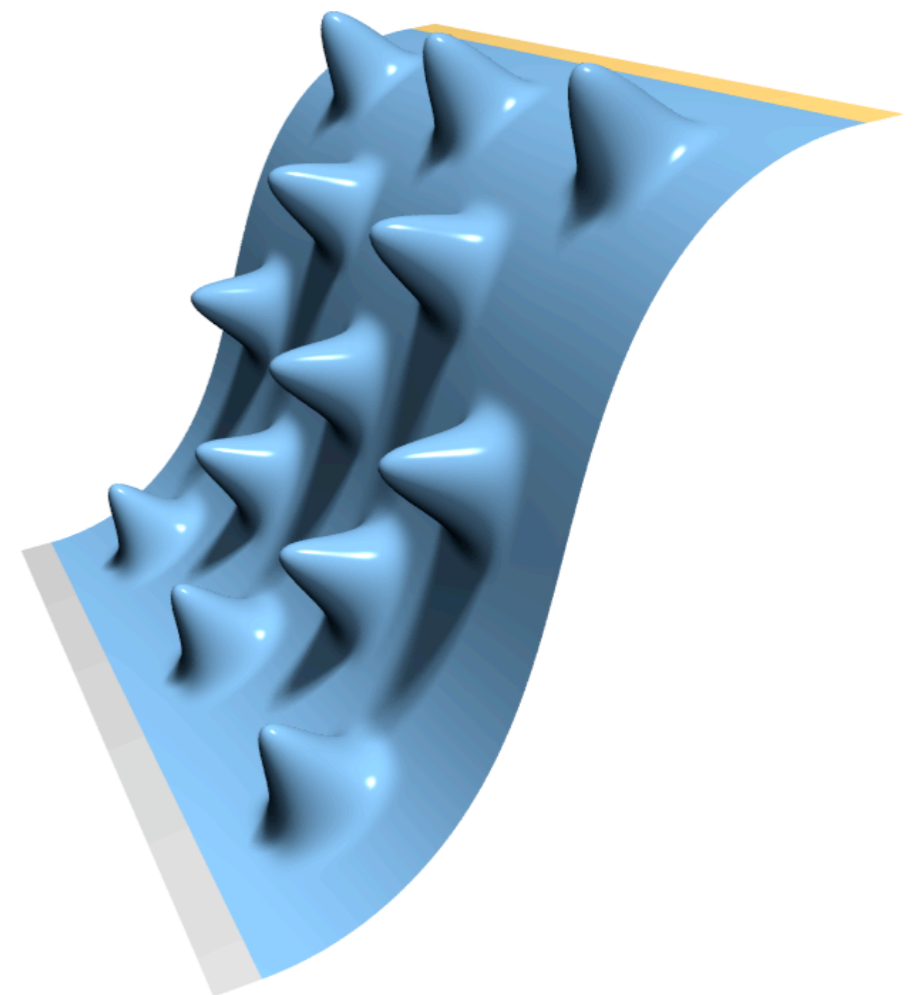
- **Boundary constraint modeling**
- Gradient-based editing
- Deformation transfer
- Equivalence
- Multiresolution Editing

# Detail-Preserving Editing

- Displacement function  $\mathbf{d}$ 
  - Smoothness
  - Detail preservation



$$\mathbf{d} : \mathcal{S} \rightarrow \mathbb{R}^3$$



# Boundary Constraint Modeling

[Botsch & Kobbelt, SIGGRAPH 04]

- Prescribe constraints

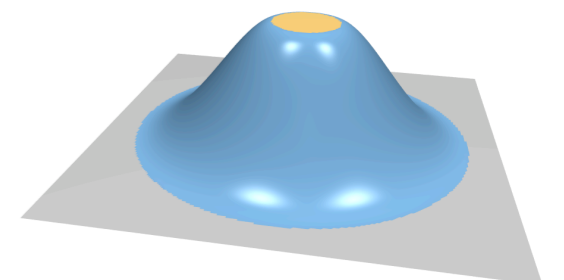
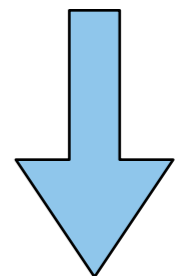
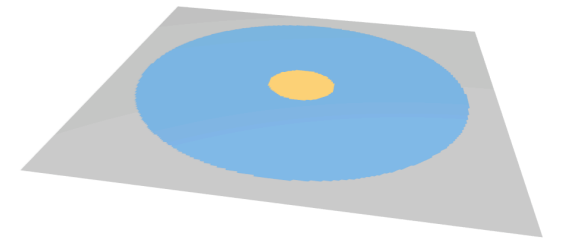
$$\mathbf{d}(\mathbf{p}_i) = \mathbf{p}'_i - \mathbf{p}_i$$

- Minimize linearized bending energy

$$\int_{\Omega} \|\mathbf{d}_{uu}\|^2 + 2\|\mathbf{d}_{uv}\|^2 + \|\mathbf{d}_{vv}\|^2 \, dudv$$

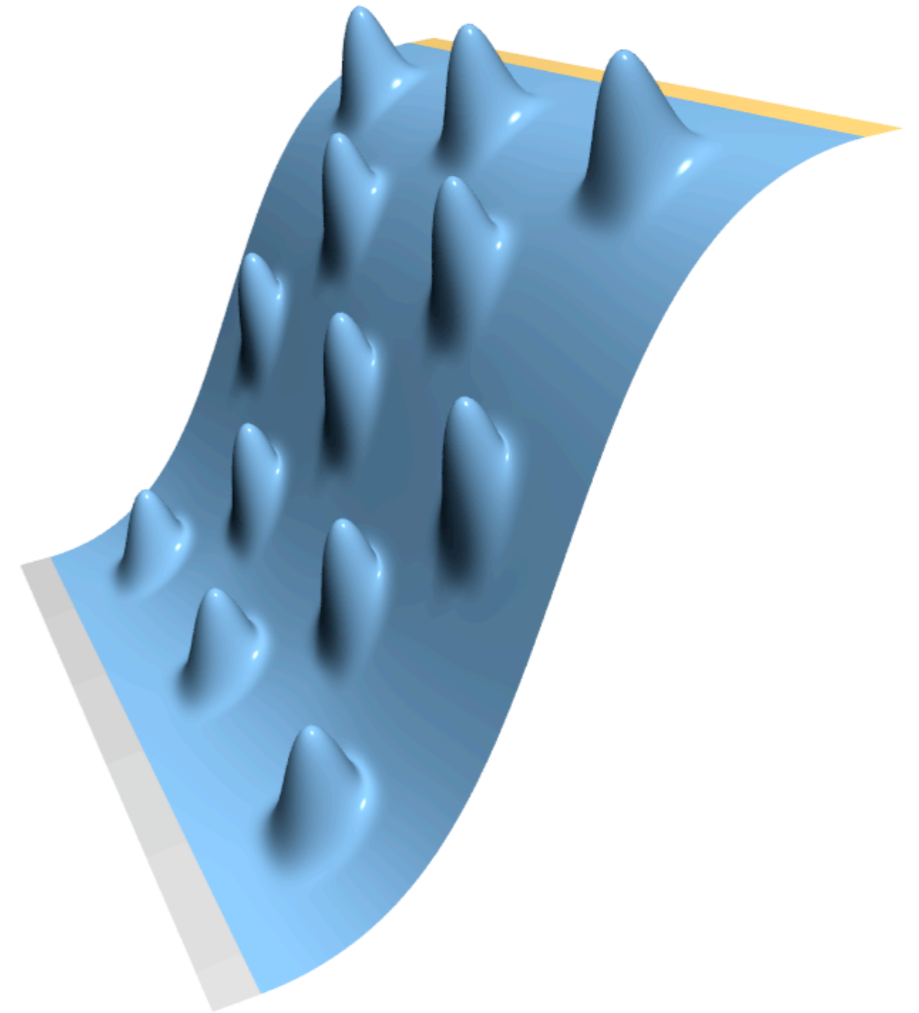
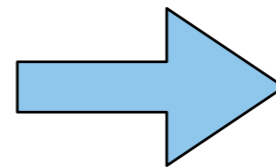
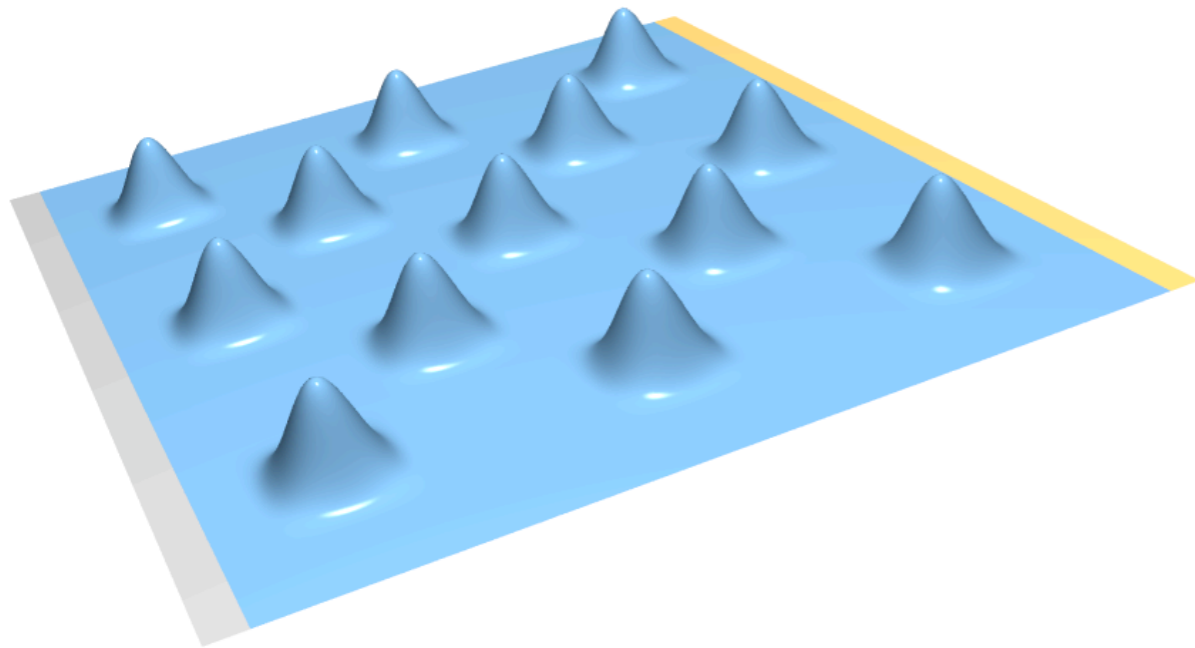
- by solving its Euler-Lagrange PDE

$$\Delta^2 \mathbf{d} = \mathbf{0}$$

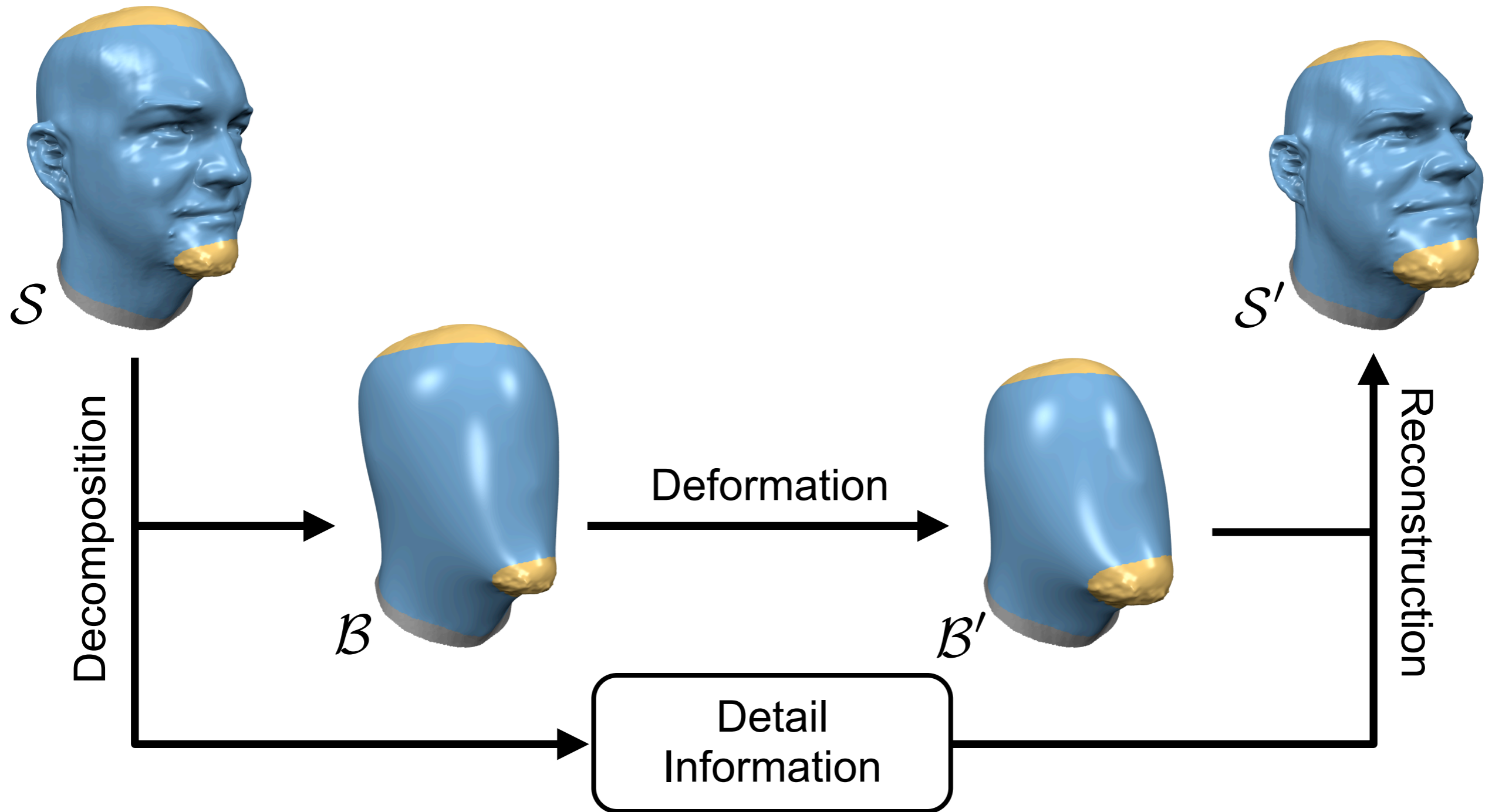


# Detail-Preserving Editing

- Linearized shell model
  - Smoothness ✓
  - Detail preservation ✗

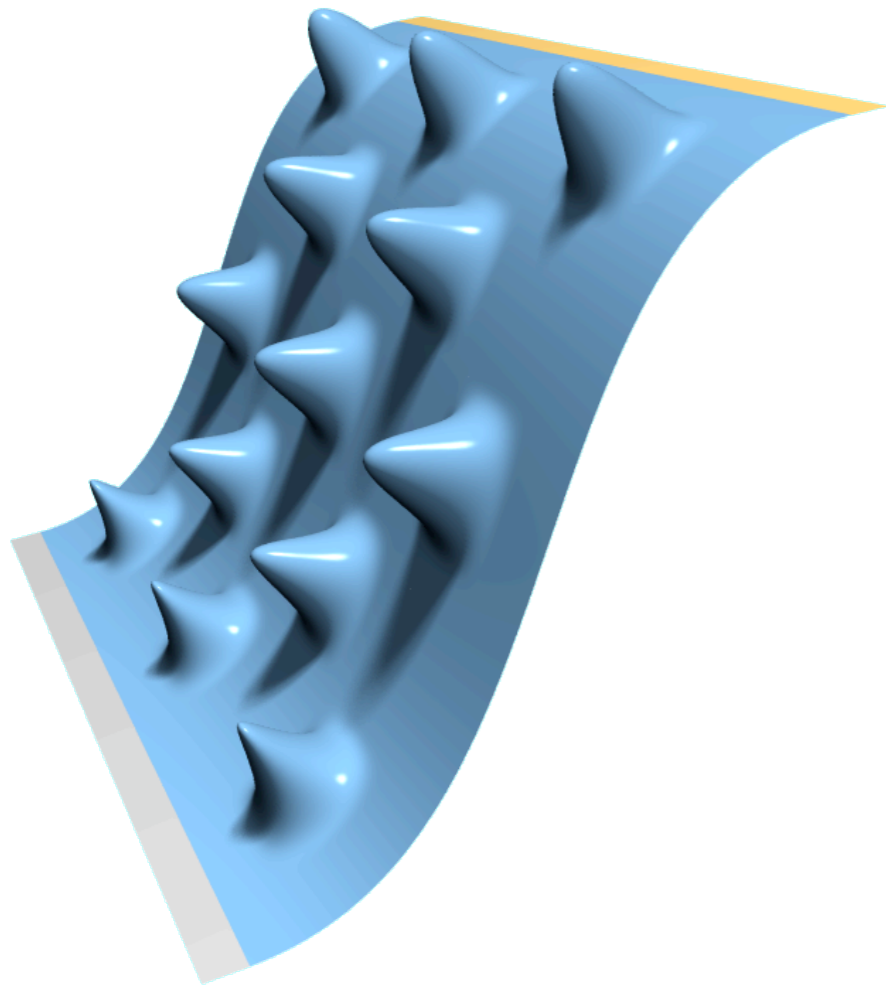


# Multiresolution Editing

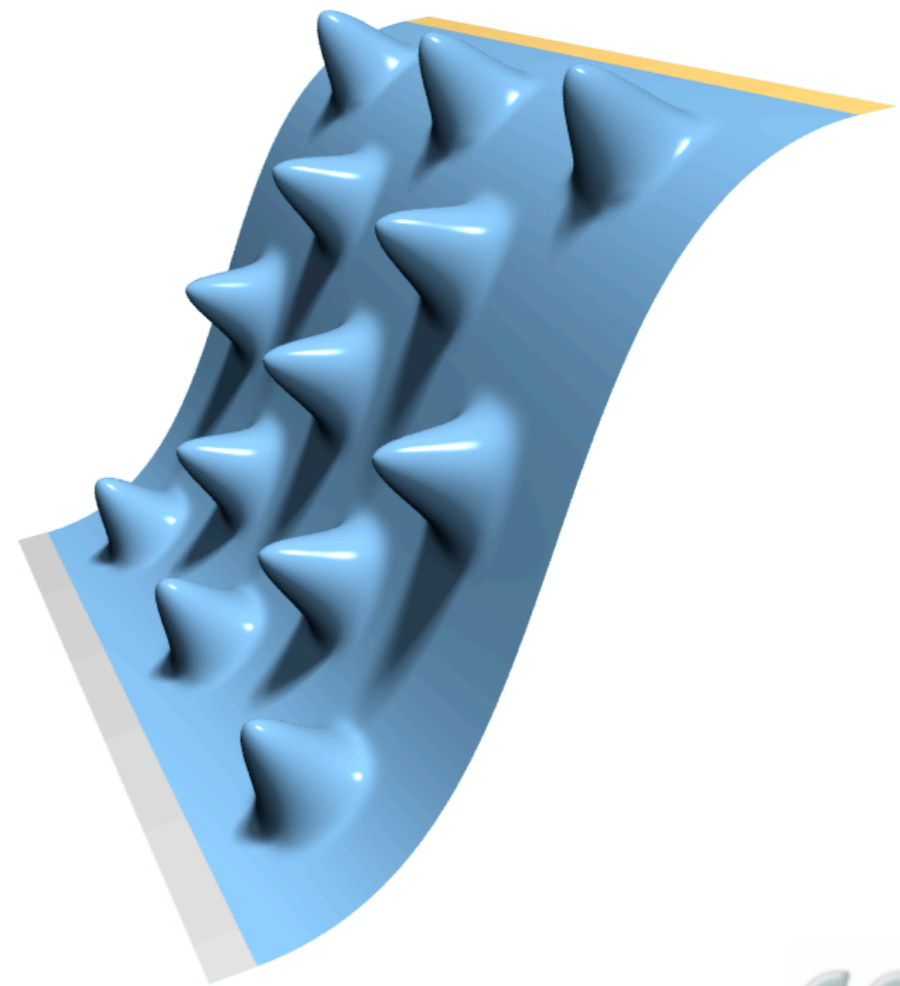


# Detail Representations

- Displacement vectors
  - very efficient
  - local self-intersections



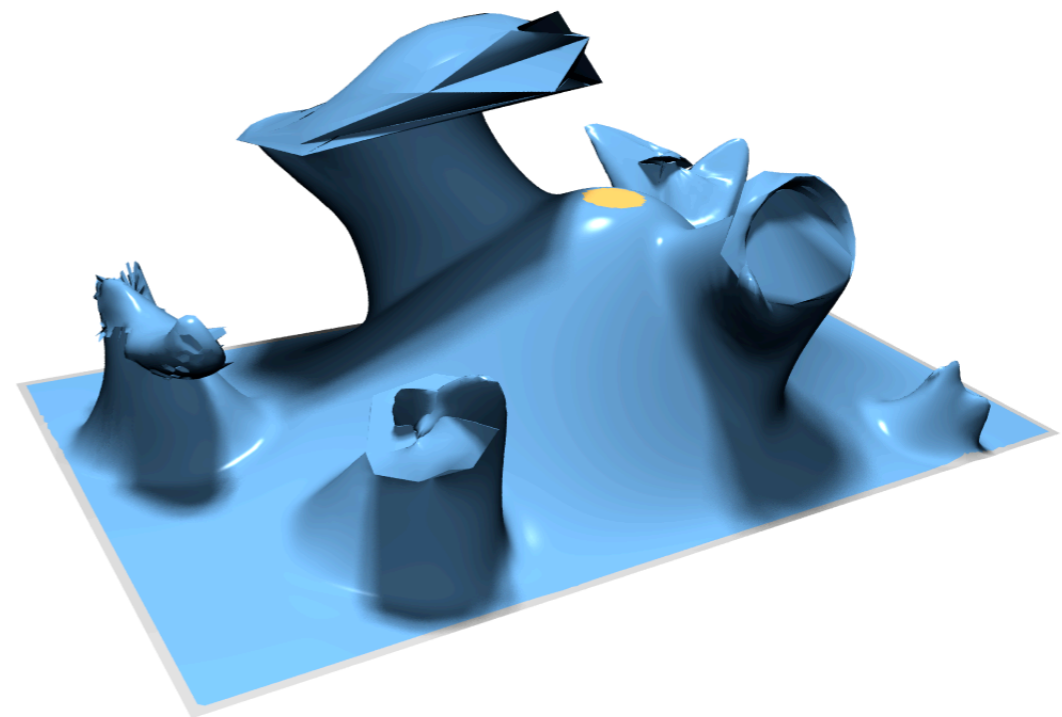
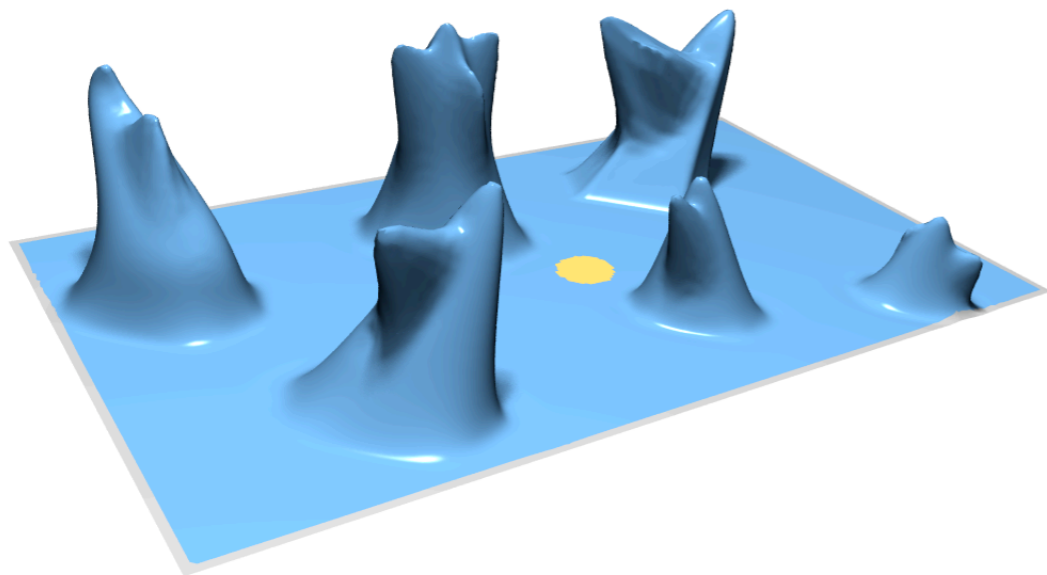
- Displacement volumes
  - avoid self-intersections
  - non-linear technique





# Complex Surfaces

- $S$  is not height field over  $B$
- Problem for displacement vectors and volumes



# Outline

- Boundary constraint modeling
- **Gradient-based editing**
- Deformation transfer
- Equivalence
- Multiresolution Editing

# Gradient-Based Editing

[Yu et al, SIGGRAPH 04]

- Gradient of coordinate function  $\mathbf{p}$ 
  - Constant per triangle  $\nabla \mathbf{p}|_{f_j} =: \mathbf{G}_j \in \mathbb{R}^{3 \times 3}$

$$\begin{pmatrix} \mathbf{G}_1 \\ \vdots \\ \mathbf{G}_F \end{pmatrix} = \underbrace{\mathbf{G}}_{\in \mathbb{R}^{3F \times V}} \cdot \begin{pmatrix} \mathbf{p}_1^T \\ \vdots \\ \mathbf{p}_V^T \end{pmatrix}$$

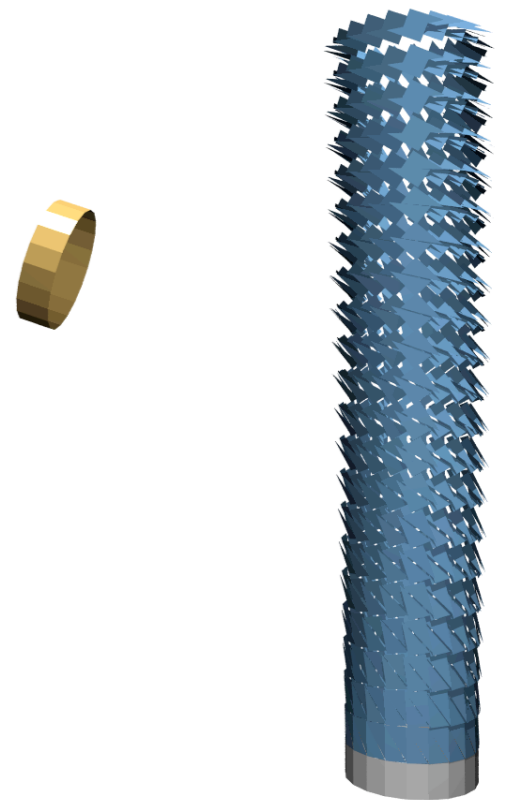


# Gradient-Based Editing

[Yu et al, SIGGRAPH 04]

- Manipulate per-face gradients
  - Gradient of deformation (rotation/scale/shear)
  - Propagate damped local rotations

$$\mathbf{G}_j \mapsto \mathbf{G}'_j$$

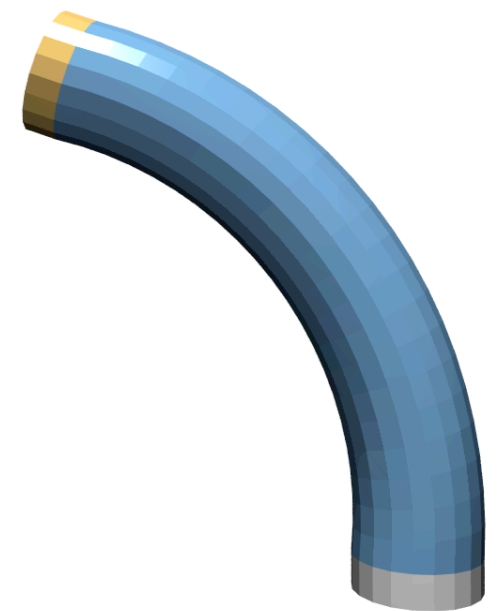


# Gradient-Based Editing

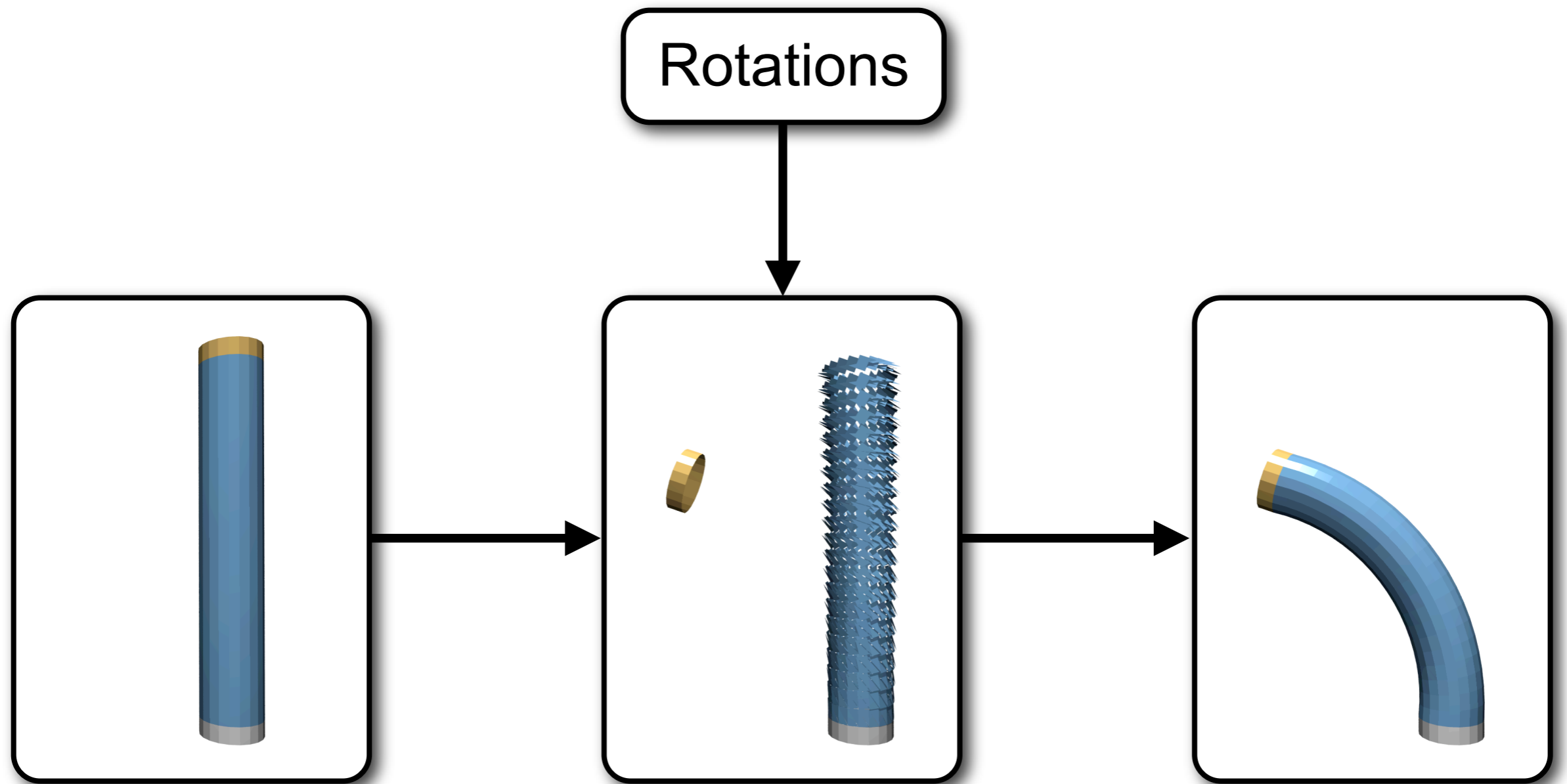
[Yu et al, SIGGRAPH 04]

- Reconstruct mesh from gradients
  - Overdetermined problem  $\mathbf{G} \in \mathbb{R}^{3F \times V}$
  - Weighted least squares system
  - Linear Poisson system

$$\text{div} \nabla = \Delta \cdot \begin{pmatrix} \mathbf{p}'_1{}^T \\ \vdots \\ \mathbf{p}'_V{}^T \end{pmatrix} = \text{div} \begin{pmatrix} \mathbf{G}'_1 \\ \vdots \\ \mathbf{G}'_F \end{pmatrix}$$

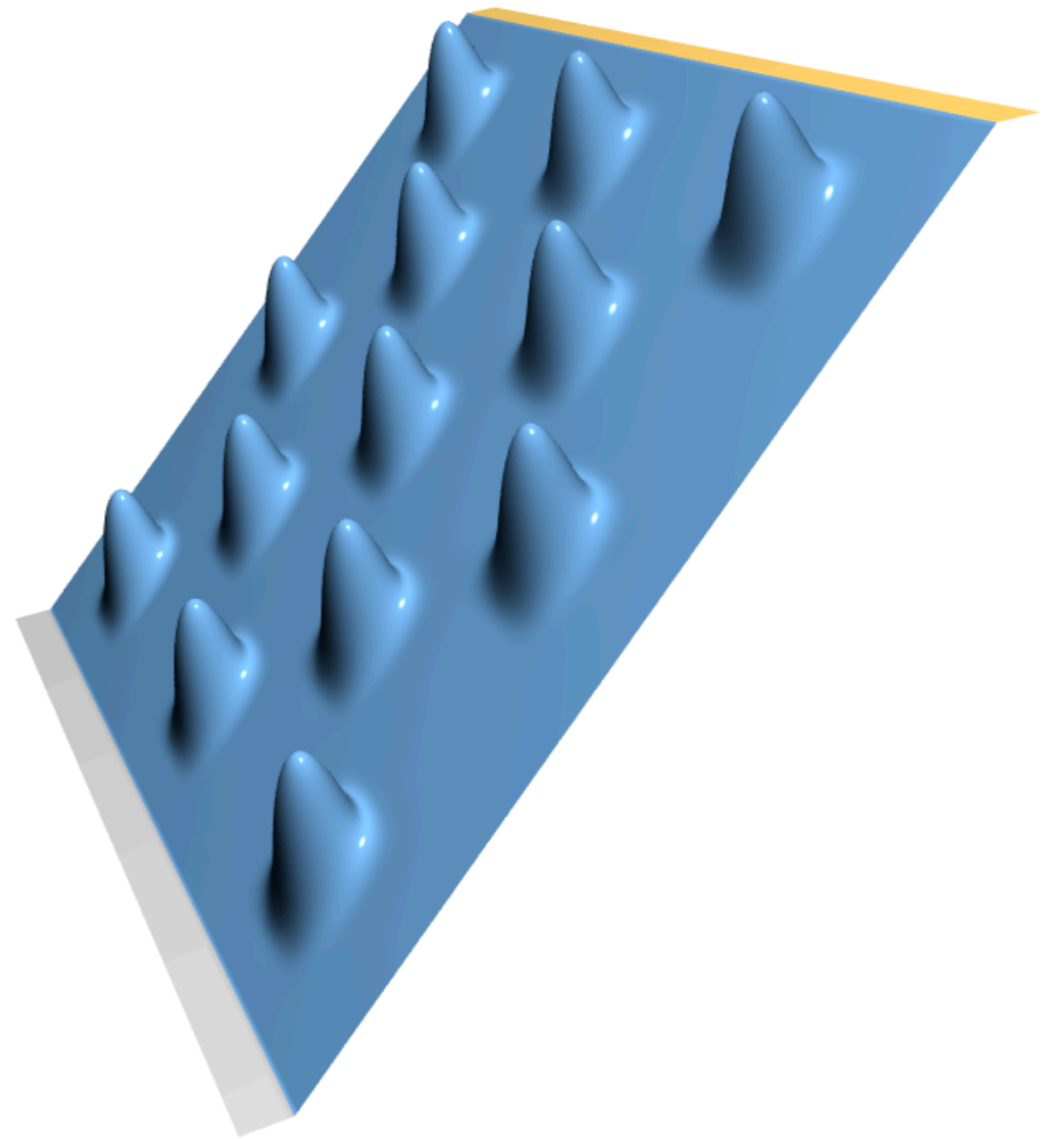


# Gradient-Based Editing



# Limitations

- Translations don't change deformation gradient
- *“Translation insensitivity”*
- $C^0$  deformation
- Detail distortion



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- Boundary constraint modeling
- Gradient-based editing
- **Deformation transfer**
- Equivalence
- Multiresolution Editing



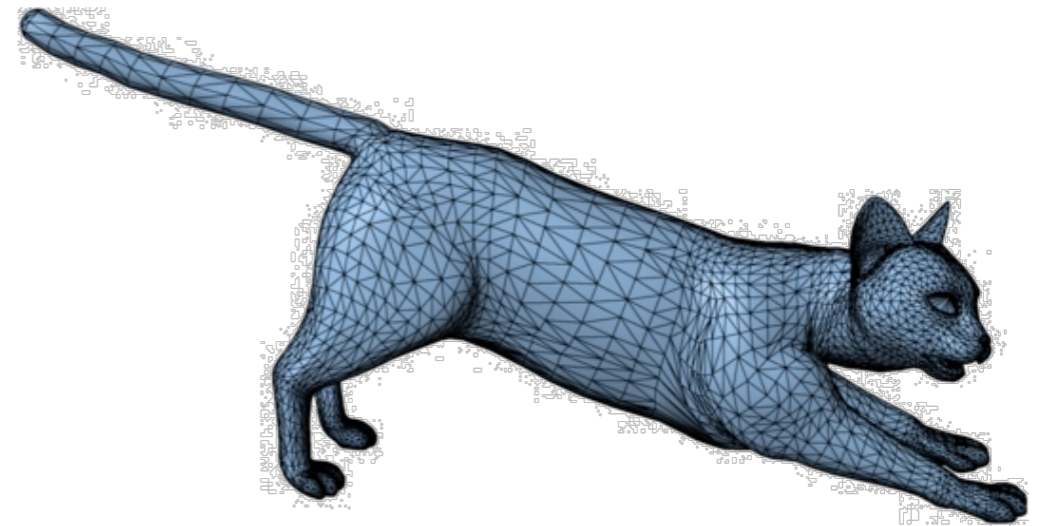
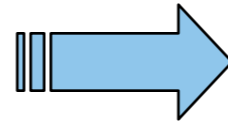
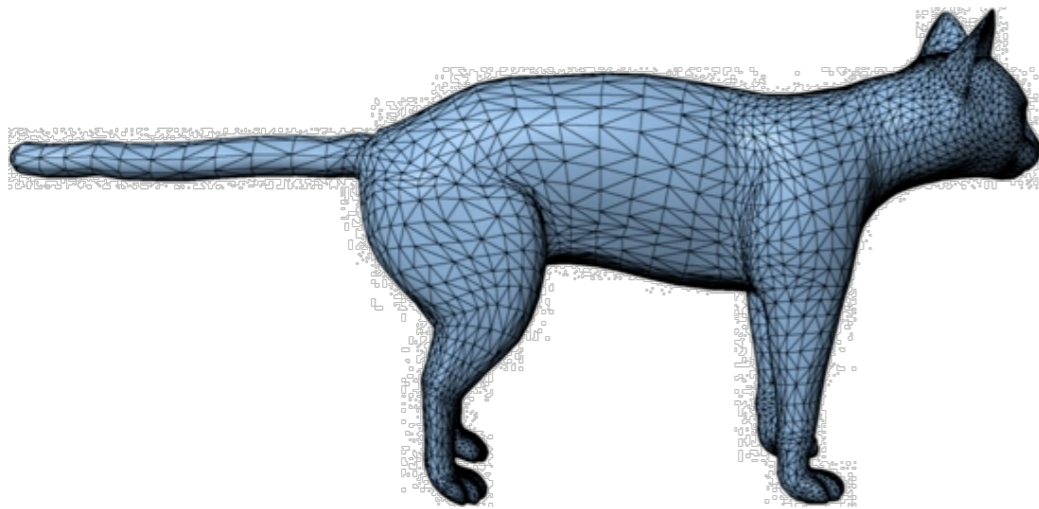
# Deformation Transfer

[Sumner & Popovic, SIGGRAPH 04]

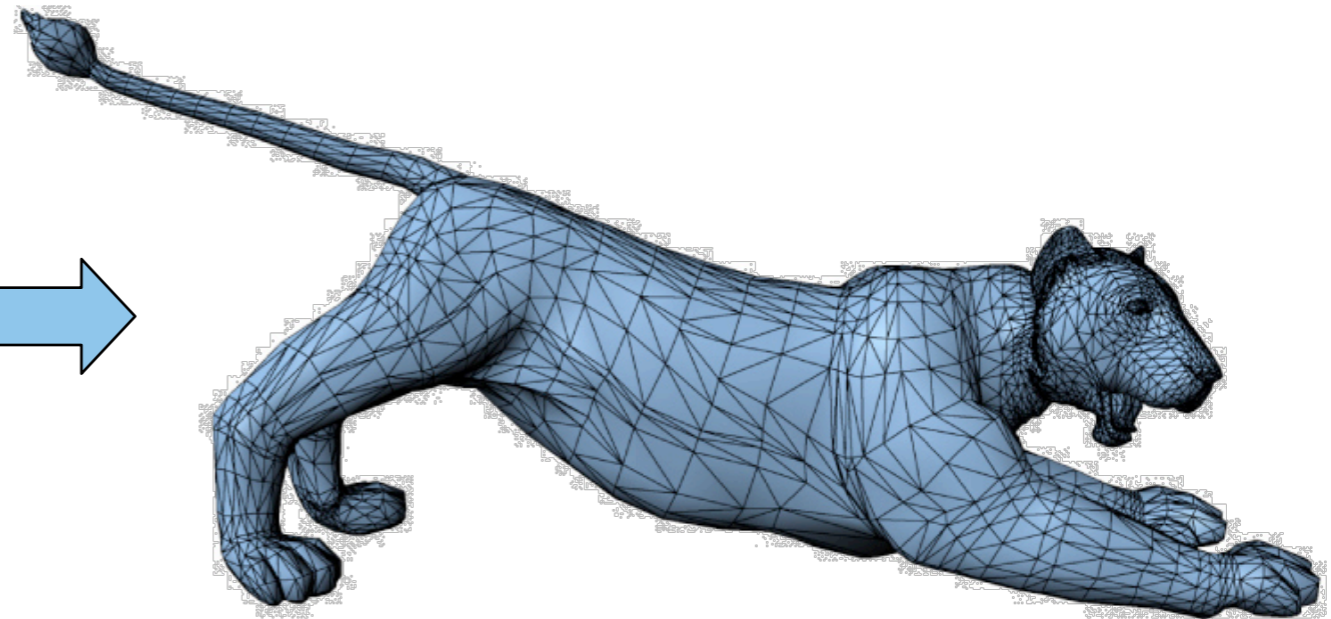
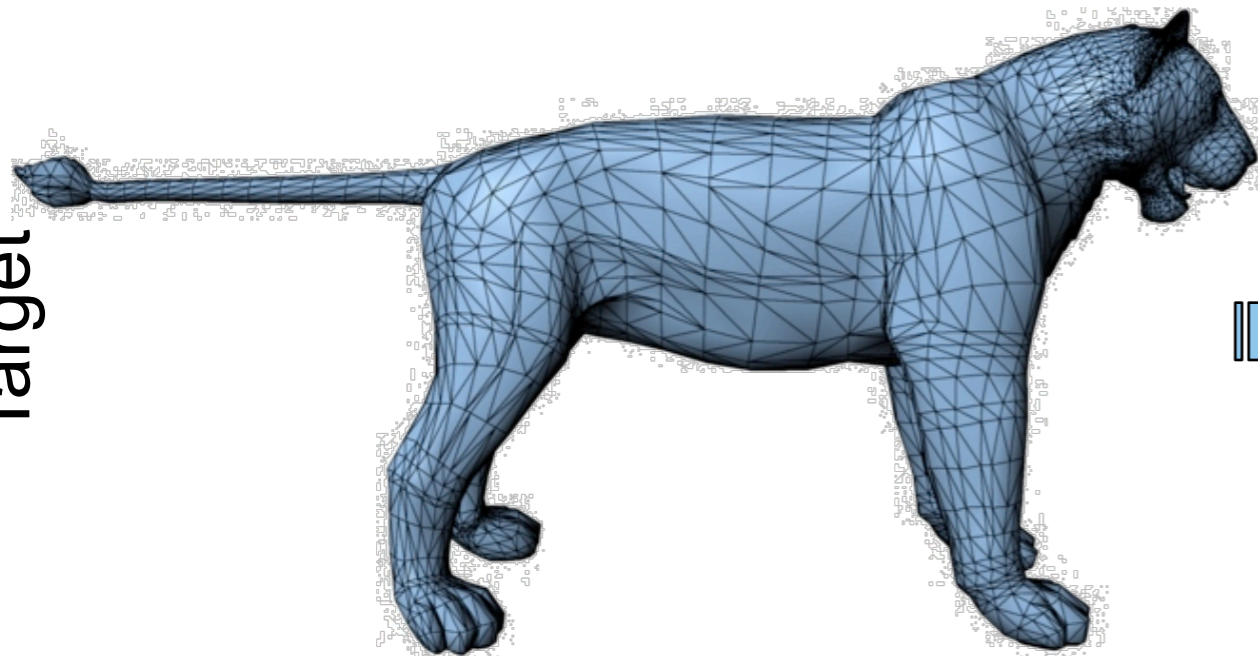
Reference

Deformed

Source



Target



# Deformation Representation

[Sumner & Popovic, SIGGRAPH 04]

- Affine transformation per triangle

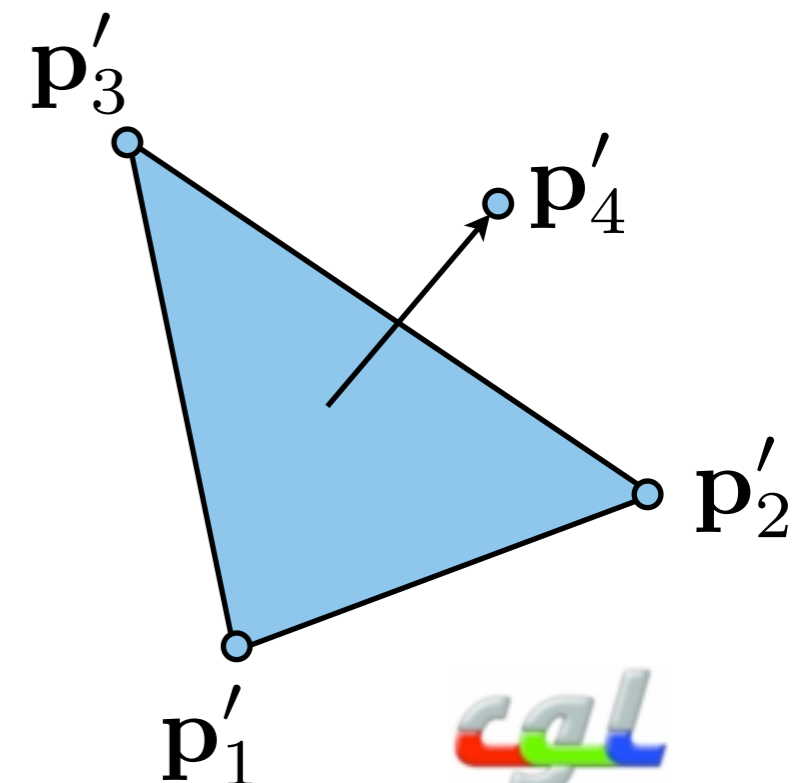
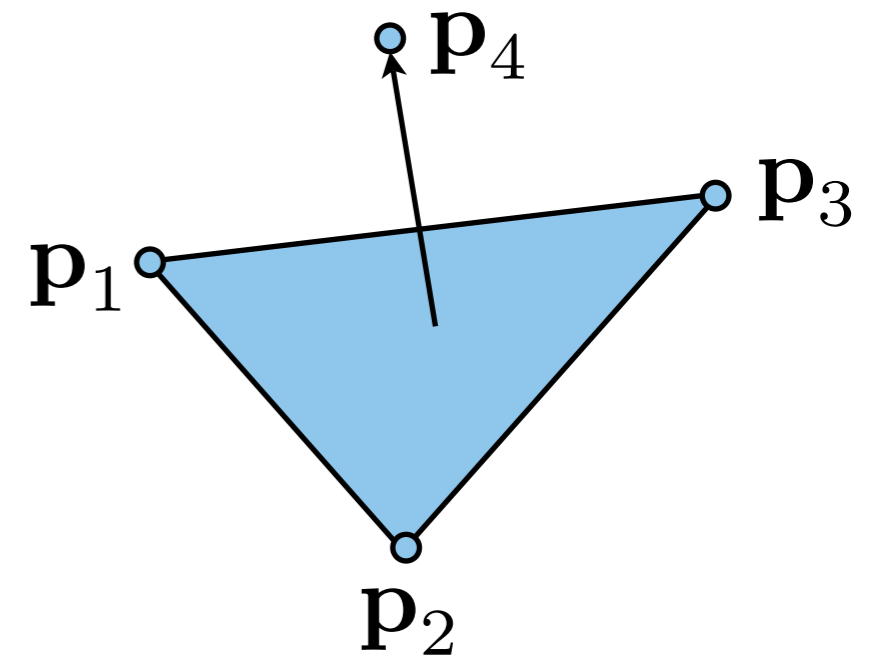
$$\mathbf{p}_i \mapsto \mathbf{T}\mathbf{p}_i + \mathbf{t}$$

- Consider deformation gradient

$$\mathbf{T} : (\mathbf{p}_1 - \mathbf{p}_4) \mapsto (\mathbf{p}'_1 - \mathbf{p}'_4)$$

$$(\mathbf{p}_2 - \mathbf{p}_4) \mapsto (\mathbf{p}'_2 - \mathbf{p}'_4)$$

$$(\mathbf{p}_3 - \mathbf{p}_4) \mapsto (\mathbf{p}'_3 - \mathbf{p}'_4)$$



# Deformation Gradients

[Sumner & Popovic, SIGGRAPH 04]

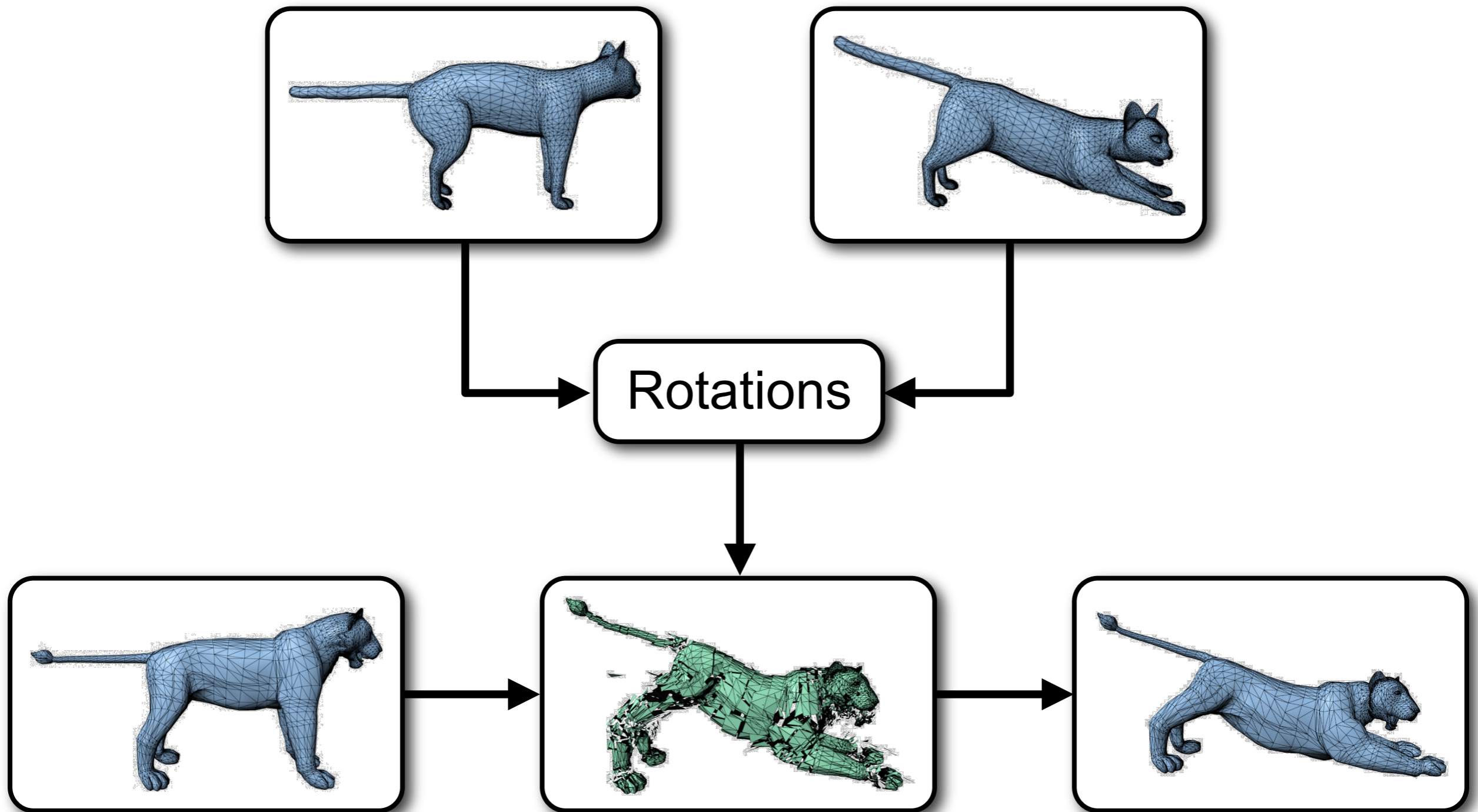
- Deformation gradients are linear in  $\mathbf{p}_i'$

$$\begin{pmatrix} \mathbf{T}_1^T \\ \vdots \\ \mathbf{T}_F^T \end{pmatrix} = \tilde{\mathbf{G}} \begin{pmatrix} \mathbf{p}'_1{}^T \\ \vdots \\ \mathbf{p}'_{V+F}{}^T \end{pmatrix}$$

- Reconstruct mesh from given def. grad.  $\mathbf{S}_i$

$$\tilde{\mathbf{G}}^T \tilde{\mathbf{G}} \begin{pmatrix} \mathbf{p}'_1{}^T \\ \vdots \\ \mathbf{p}'_{V+F}{}^T \end{pmatrix} = \tilde{\mathbf{G}}^T \begin{pmatrix} \mathbf{S}_1^T \\ \vdots \\ \mathbf{S}_F^T \end{pmatrix}$$

# Deformation Transfer



# Outline

- Boundary constraint modeling
- Gradient-based editing
- Deformation transfer
- **Equivalence**
- Multiresolution Editing

# Equivalence

## Gradient-Based Editing

$$\mathbf{G}^T \mathbf{D} \mathbf{G} \begin{pmatrix} \mathbf{p}'_1{}^T \\ \vdots \\ \mathbf{p}'_V{}^T \end{pmatrix} = \mathbf{G}^T \mathbf{D} \begin{pmatrix} \mathbf{G}'_1 \\ \vdots \\ \mathbf{G}'_F \end{pmatrix}$$

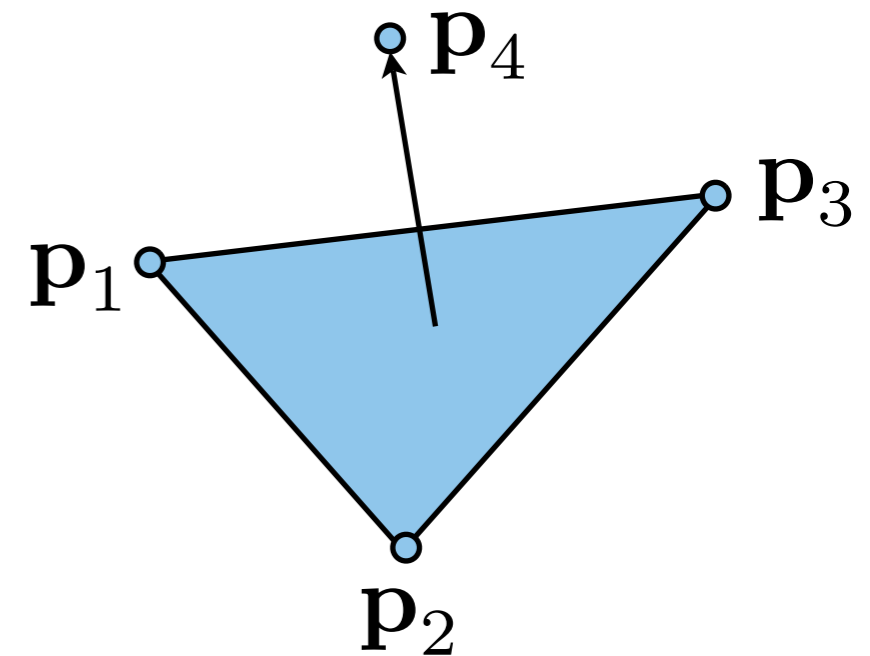
## Deformation Transfer

$$\tilde{\mathbf{G}}^T \tilde{\mathbf{G}} \begin{pmatrix} \mathbf{p}'_1{}^T \\ \vdots \\ \mathbf{p}'_{V+F}{}^T \end{pmatrix} = \tilde{\mathbf{G}}^T \begin{pmatrix} \mathbf{S}_1^T \\ \vdots \\ \mathbf{S}_F^T \end{pmatrix}$$

# Deformation Representation

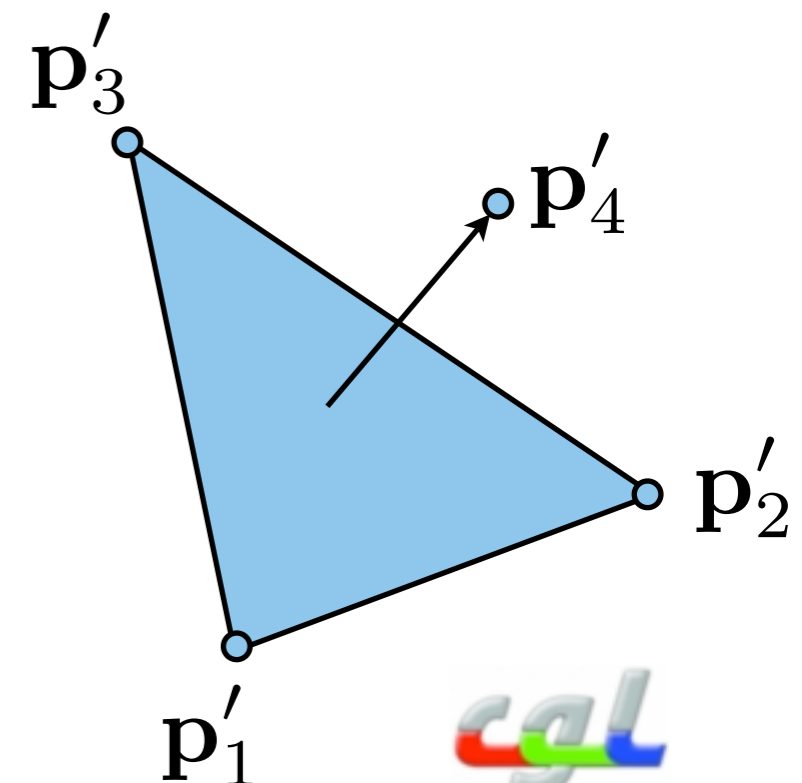
- 4-point deformation gradient

$$\begin{aligned} \mathbf{T} : (\mathbf{p}_1 - \mathbf{p}_4) &\mapsto (\mathbf{p}'_1 - \mathbf{p}'_4) \\ (\mathbf{p}_2 - \mathbf{p}_4) &\mapsto (\mathbf{p}'_2 - \mathbf{p}'_4) \\ (\mathbf{p}_3 - \mathbf{p}_4) &\mapsto (\mathbf{p}'_3 - \mathbf{p}'_4) \end{aligned}$$



- 3-points plus normal

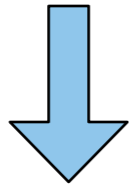
$$\begin{aligned} \mathbf{T} : (\mathbf{p}_1 - \mathbf{p}_3) &\mapsto (\mathbf{p}'_1 - \mathbf{p}'_3) \\ (\mathbf{p}_2 - \mathbf{p}_3) &\mapsto (\mathbf{p}'_2 - \mathbf{p}'_3) \\ \mathbf{n} &\mapsto \mathbf{0} \end{aligned}$$



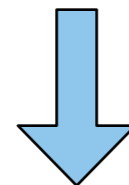
# Equivalence

## Gradient-Based Editing

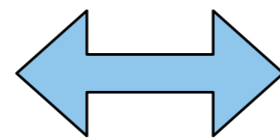
$$\mathbf{G}^T \mathbf{D} \mathbf{G} \begin{pmatrix} \mathbf{p}'_1{}^T \\ \vdots \\ \mathbf{p}'_V{}^T \end{pmatrix} = \mathbf{G}^T \mathbf{D} \begin{pmatrix} \mathbf{G}'_1 \\ \vdots \\ \mathbf{G}'_F \end{pmatrix}$$



$$\mathbf{G}^T \mathbf{D} \mathbf{G} \begin{pmatrix} \mathbf{p}'_1{}^T \\ \vdots \\ \mathbf{p}'_V{}^T \end{pmatrix} = \mathbf{G}^T \mathbf{D} \begin{pmatrix} \mathbf{G}_1 \mathbf{S}_1^T \\ \vdots \\ \mathbf{G}'_F \mathbf{S}_F^T \end{pmatrix}$$

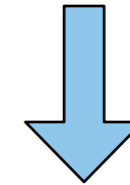


$$\mathbf{G}^T \mathbf{D} \mathbf{G} \begin{pmatrix} \mathbf{p}'_1{}^T \\ \vdots \\ \mathbf{p}'_V{}^T \end{pmatrix} = \mathbf{G}^T \mathbf{D} \begin{pmatrix} \mathbf{S}_1^T \\ \vdots \\ \mathbf{S}_F^T \end{pmatrix}$$

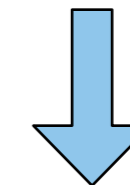


## Deformation Transfer

$$\tilde{\mathbf{G}}^T \tilde{\mathbf{G}} \begin{pmatrix} \mathbf{p}'_1{}^T \\ \vdots \\ \mathbf{p}'_{V+F}{}^T \end{pmatrix} = \tilde{\mathbf{G}}^T \begin{pmatrix} \mathbf{S}_1^T \\ \vdots \\ \mathbf{S}_F^T \end{pmatrix}$$



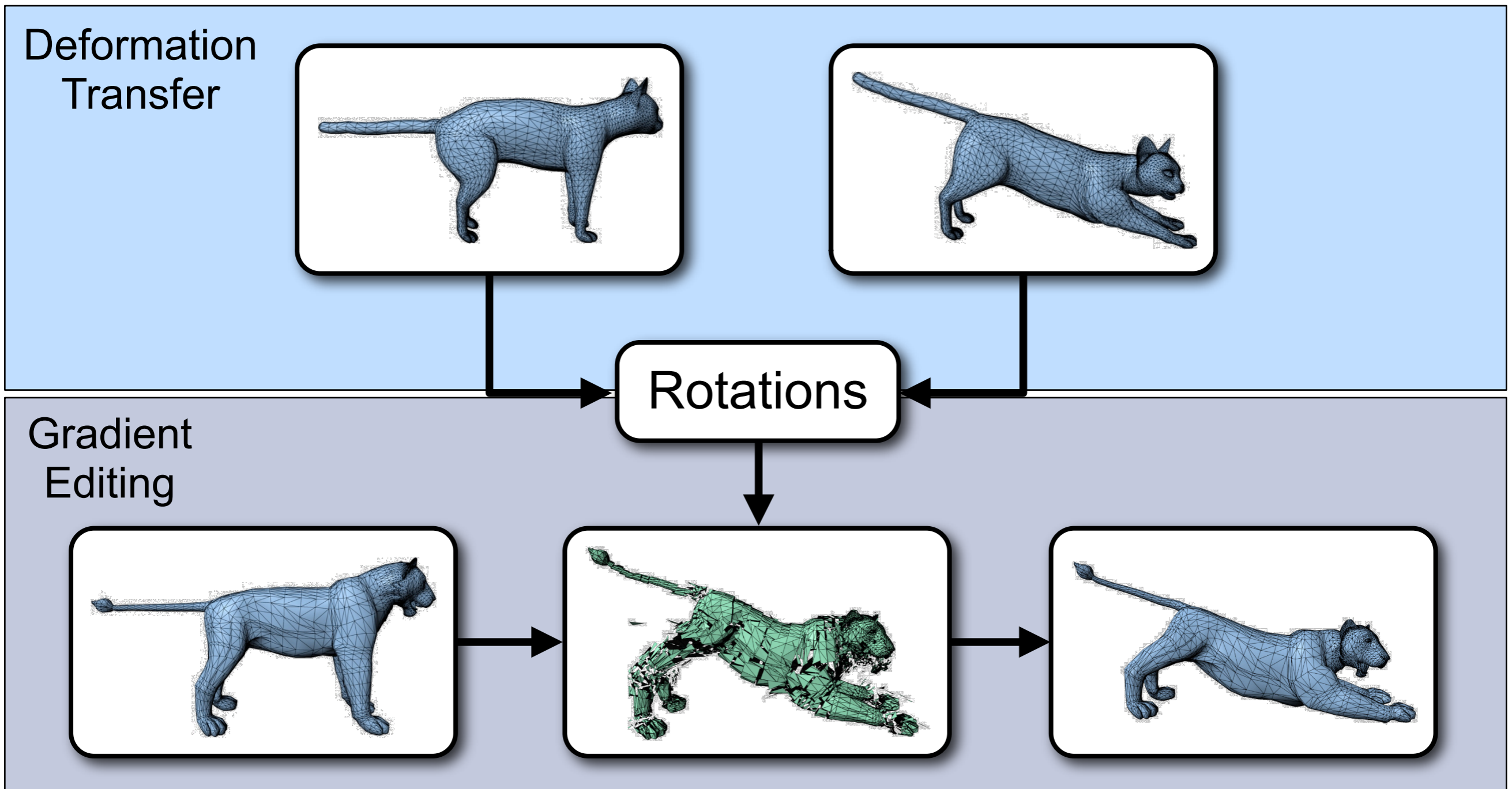
$$\mathbf{G}^T \mathbf{G} \begin{pmatrix} \mathbf{p}'_1{}^T \\ \vdots \\ \mathbf{p}'_V{}^T \end{pmatrix} = \mathbf{G}^T \begin{pmatrix} \mathbf{S}_1^T \\ \vdots \\ \mathbf{S}_F^T \end{pmatrix}$$



$$\mathbf{G}^T \mathbf{D} \mathbf{G} \begin{pmatrix} \mathbf{p}'_1{}^T \\ \vdots \\ \mathbf{p}'_V{}^T \end{pmatrix} = \mathbf{G}^T \mathbf{D} \begin{pmatrix} \mathbf{S}_1^T \\ \vdots \\ \mathbf{S}_F^T \end{pmatrix}$$



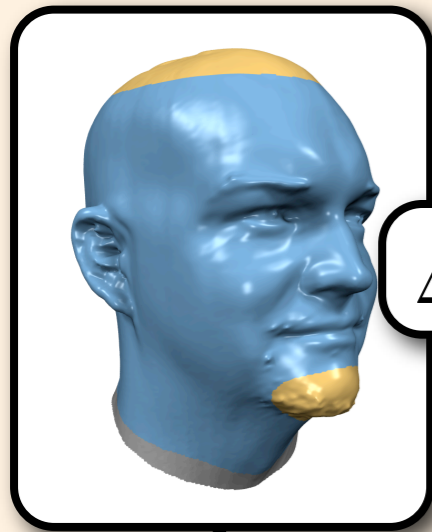
# Deformation Transfer



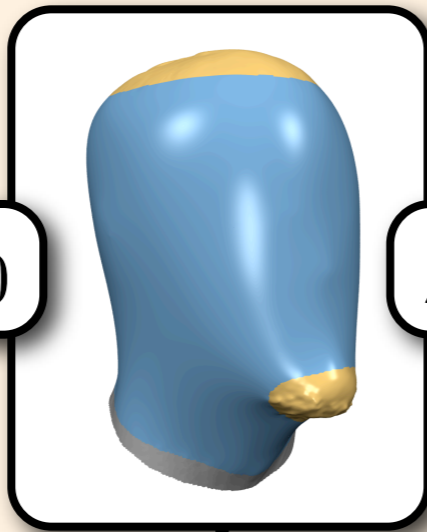
# Outline

- Boundary constraint modeling
- Gradient-based editing
- Deformation transfer
- Equivalence
- **Multiresolution Editing**

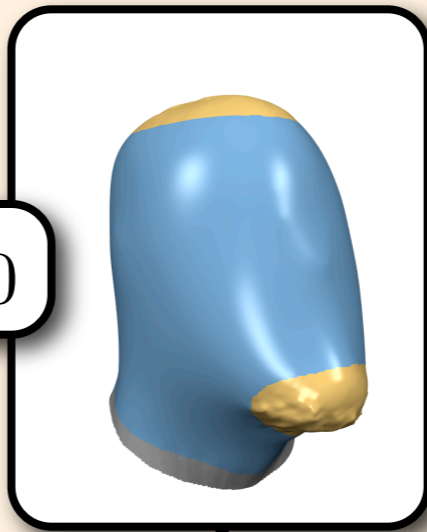
# Boundary Constraint Modeling



$$\Delta^2 \mathbf{b} = 0$$



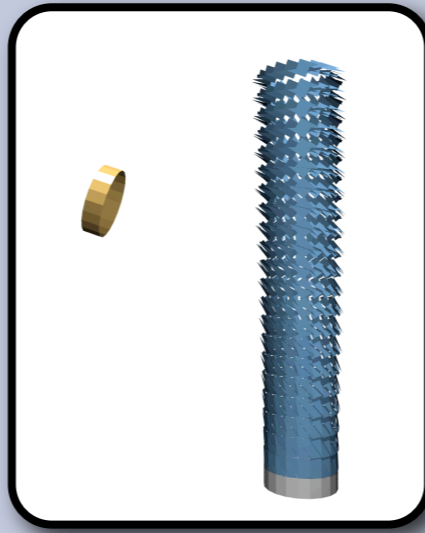
$$\Delta^2 \mathbf{d} = 0$$



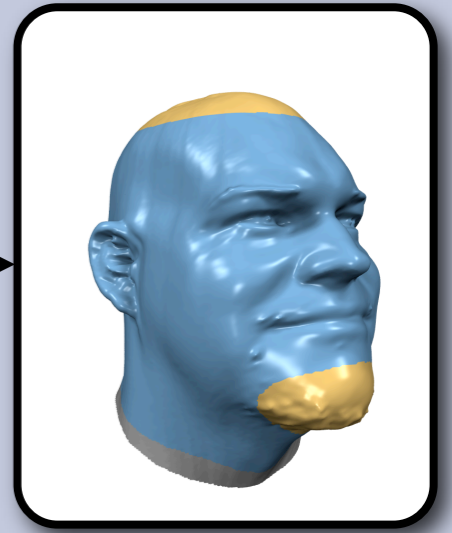
Deform.  
Transfer

Rotations

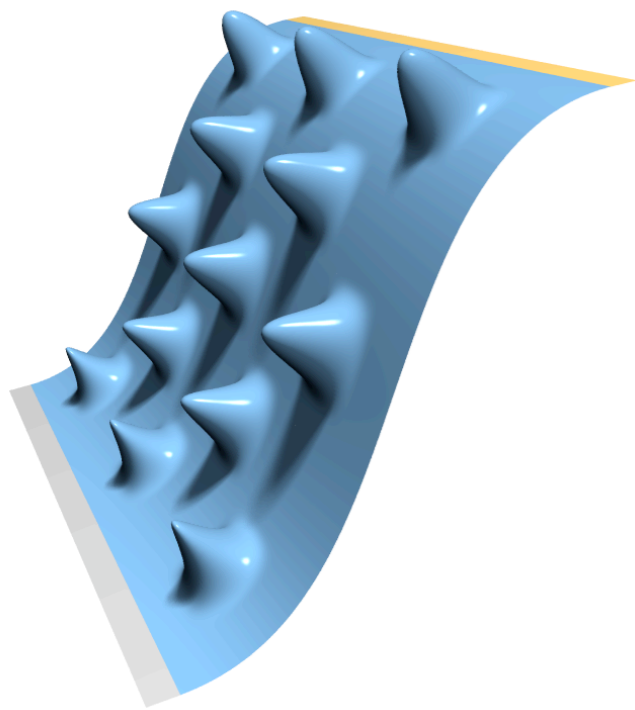
Gradient  
Editing



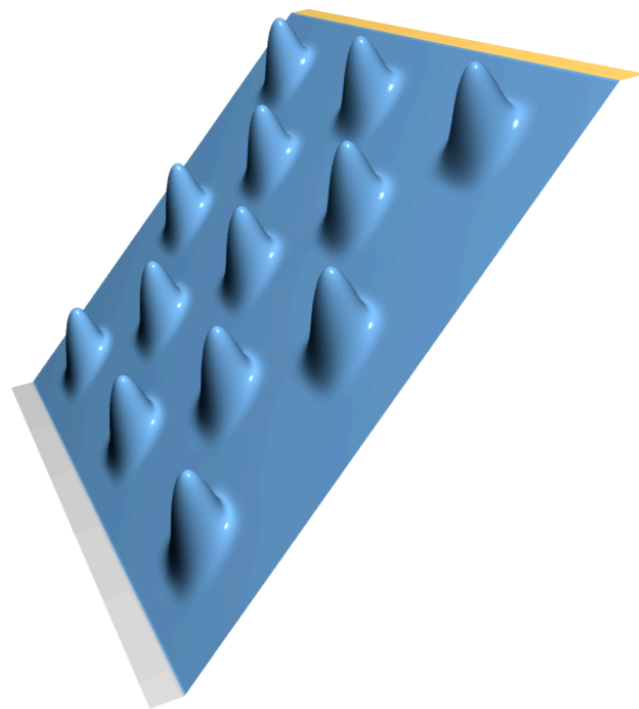
$$\Delta \mathbf{p} = \nabla \cdot \mathbf{G}'$$



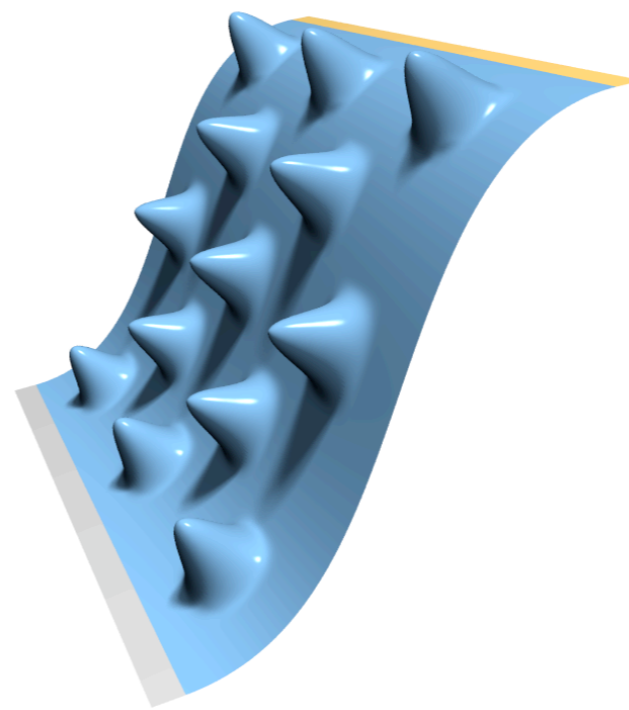
# Pure Translation



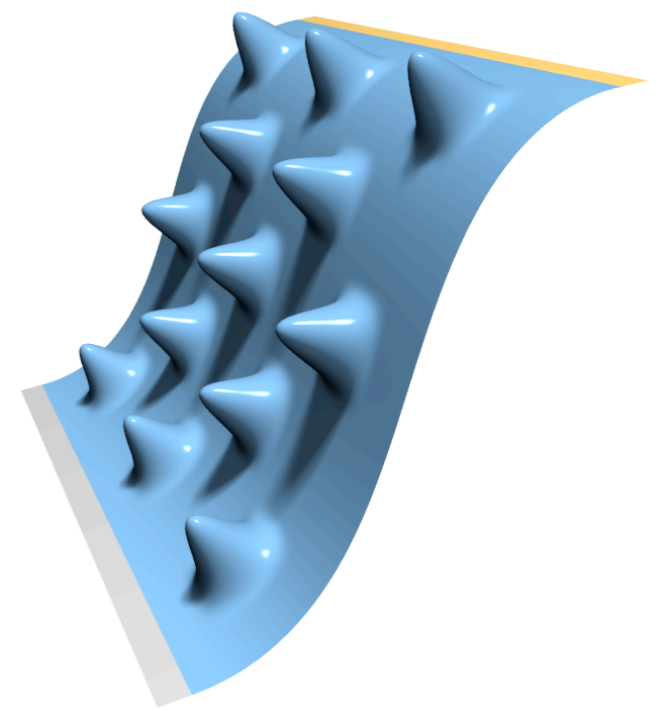
Displacement  
Vectors



Gradient  
Editing

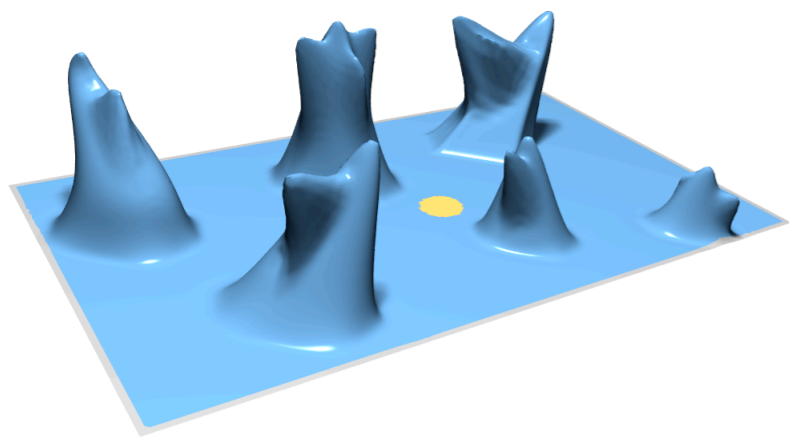


Displacement  
Volumes

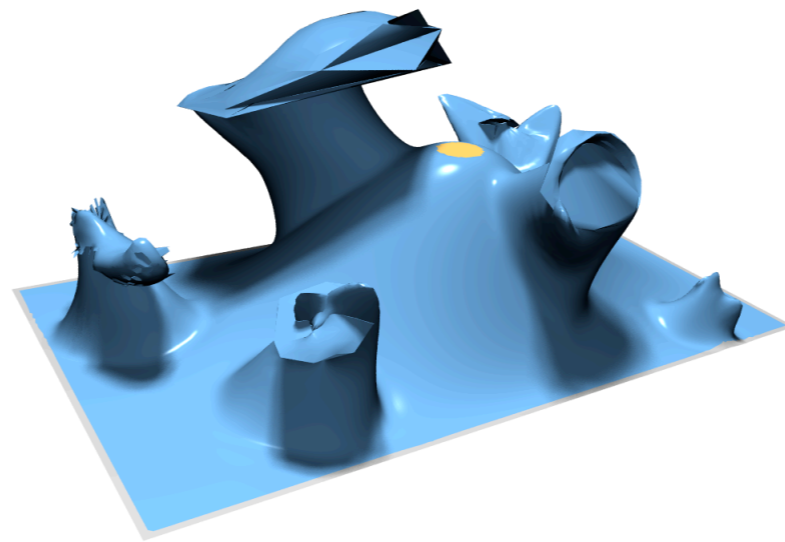


Deformation  
Transfer

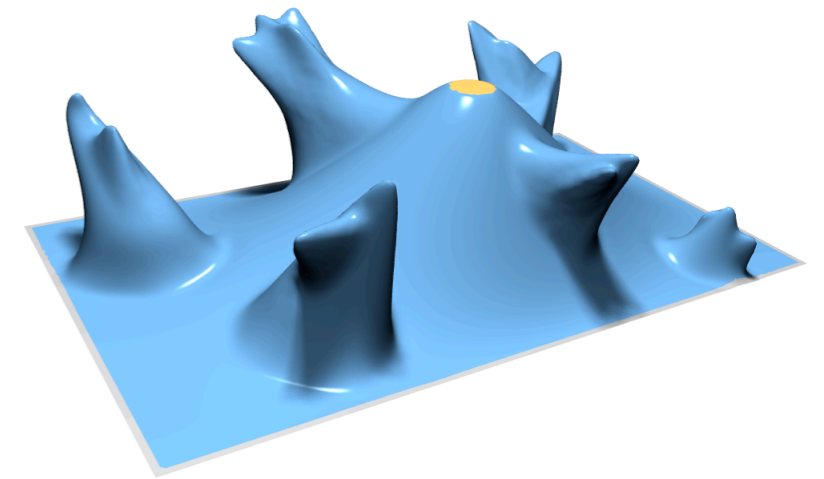
# Non Height Field



Original



Displacement  
Vectors



Deformation  
Transfer

# Conclusion

- Deformation Transfer == Gradient-Based Editing
  - Improve deformation transfer by factor 3
- New multiresolution representation
  - No height field restriction
  - Avoids self-intersections
  - Still linear and efficient
- Define rotations for gradients
  - Not translation-insensitive