PriMo: Coupled Prisms for Intuitive Surface Modeling

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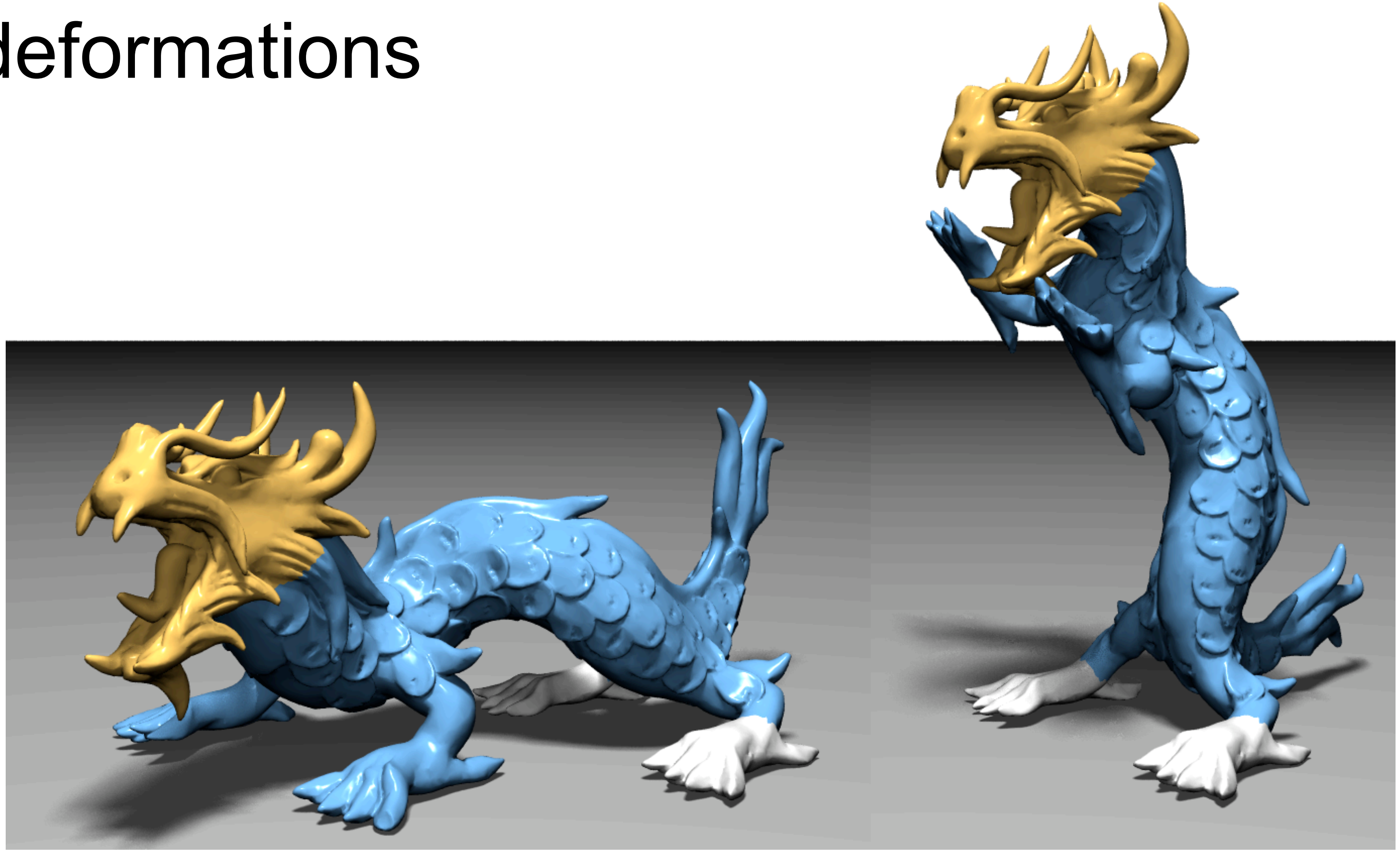
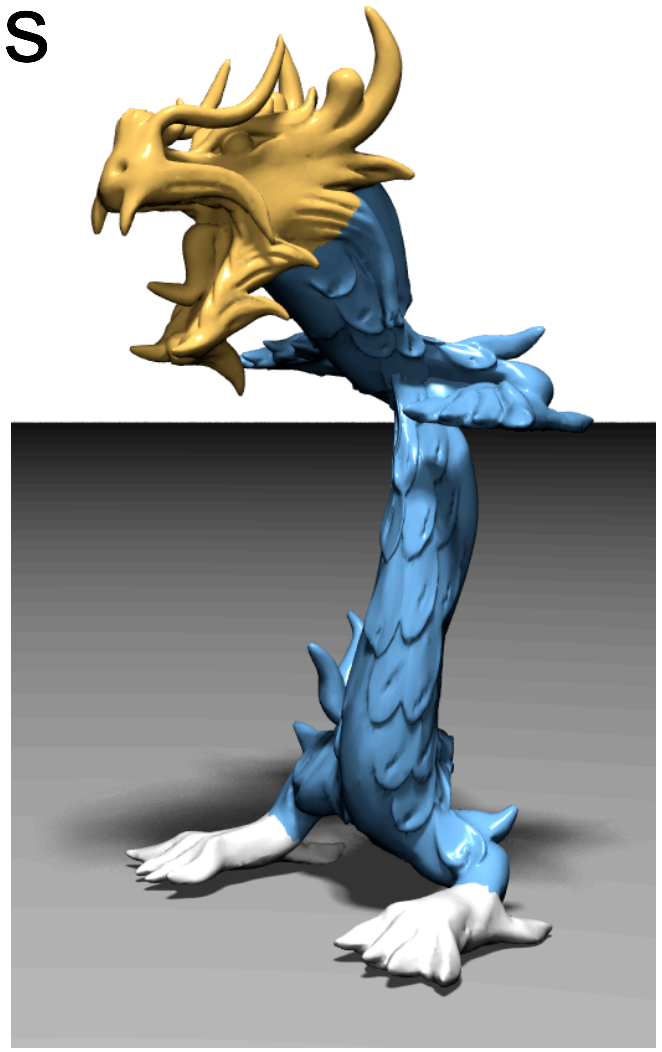
Surface Deformation

- Requirements
  - Easy and intuitive user interaction
  - Large-scale deformations
  - Robustness
  - Efficiency

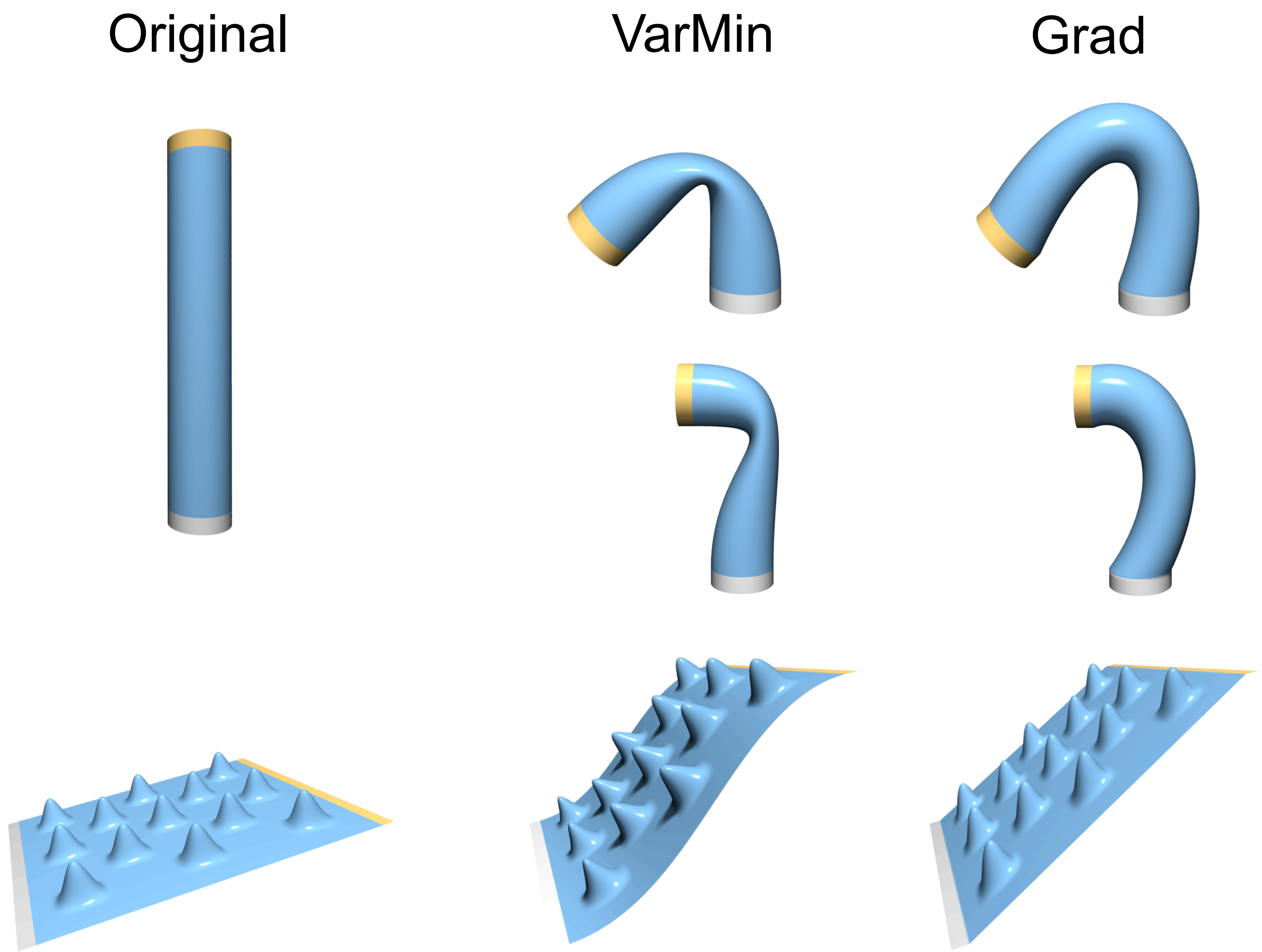


Surface Deformation

- Recent methods focus more on efficiency
  - Real-time deformations of large models
- Requires linearization
  - Problems with large deformations
- Split large deformations
  - Specify more constraints
  - More user guidance required



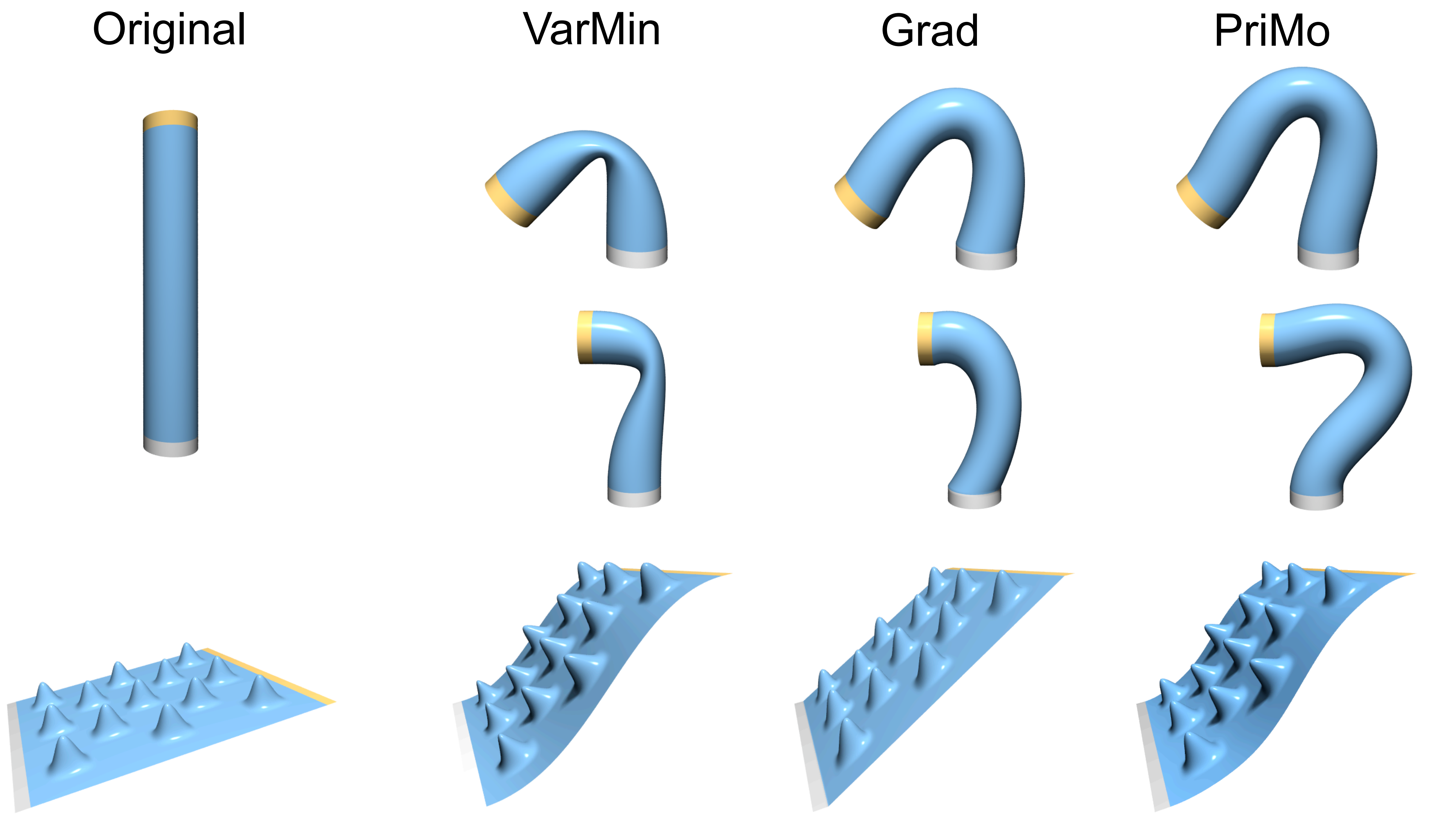
Linear Techniques



Non-Linear Surface Deformation

- Use a non-linear deformation model
  - Too slow, complicated, instable?
- Physically *plausible* vs. physically *correct*
- Trade physical correctness for
  - Computational efficiency
  - Numerical robustness

Comparison



Outline

- Motivation
- **Prism Representation**
- Geometric Optimization
- Results

Elastically Connected Rigid Prisms

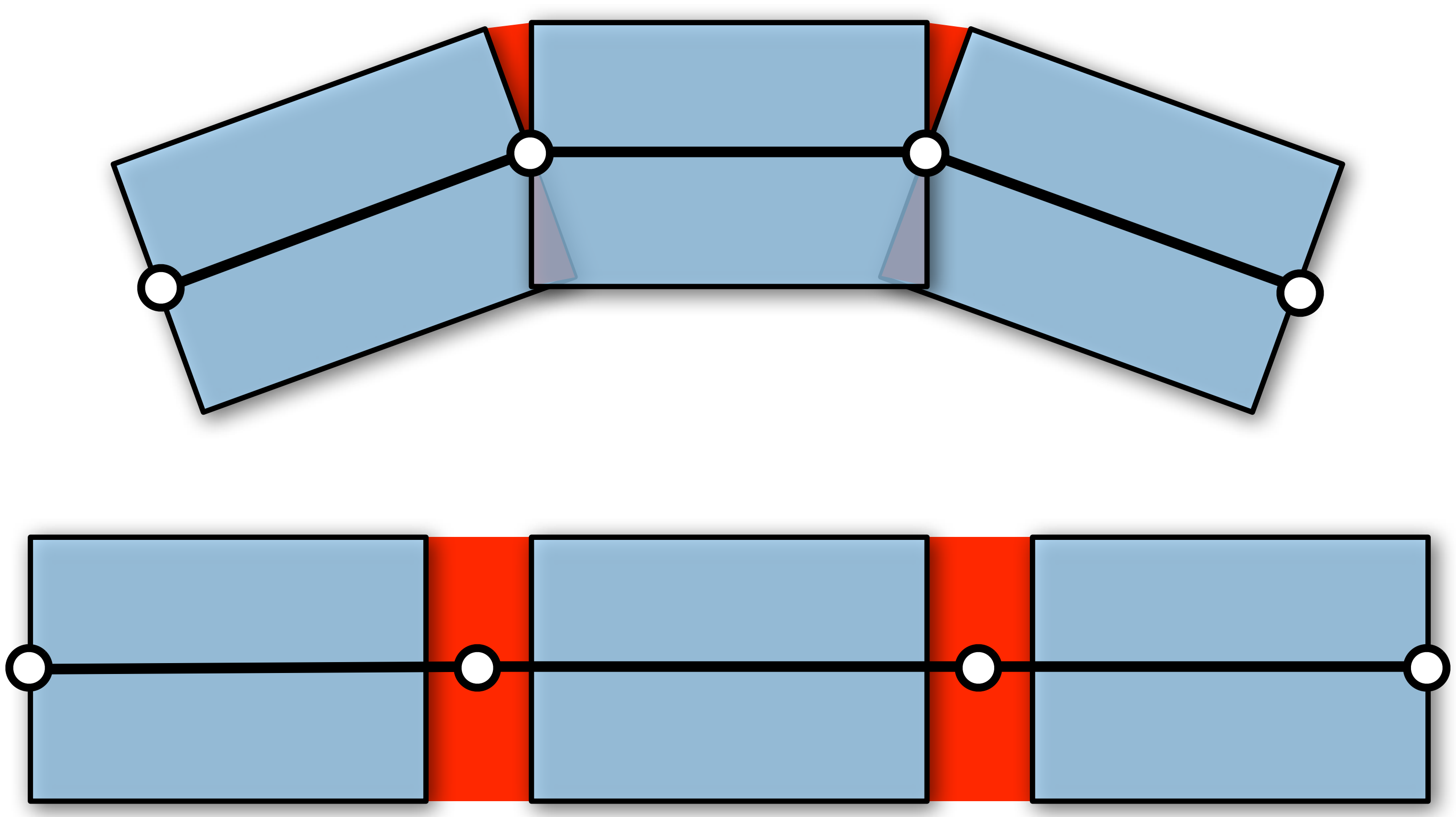
- Qualitatively emulate thin-shell behavior
- Thin volumetric layer around center surface
- Extrude polygonal prism P_i per mesh face F_i

Elastically Connected Rigid Prisms

- How to deform prisms?
  – FEM has problems if elements degenerate...
- Prevent prisms from degenerating
  ➡ Keep them *rigid*

Elastically Connected Rigid Prisms

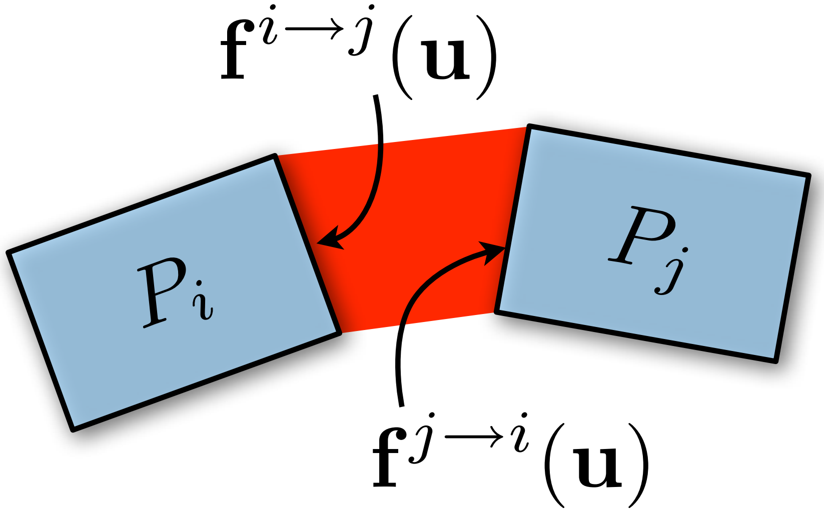
- Connect prisms along their faces
  - Non-linear elastic energy
  - Measures bending, stretching, twisting, ...



Elastically Connected Rigid Prisms

- Pairwise prism energy

$$E_{ij} = \int_{[0,1]^2} \left\| \mathbf{f}^{\,i\rightarrow j}(\mathbf{u}) - \mathbf{f}^{\,j\rightarrow i}(\mathbf{u}) \right\|^2 d\mathbf{u}$$



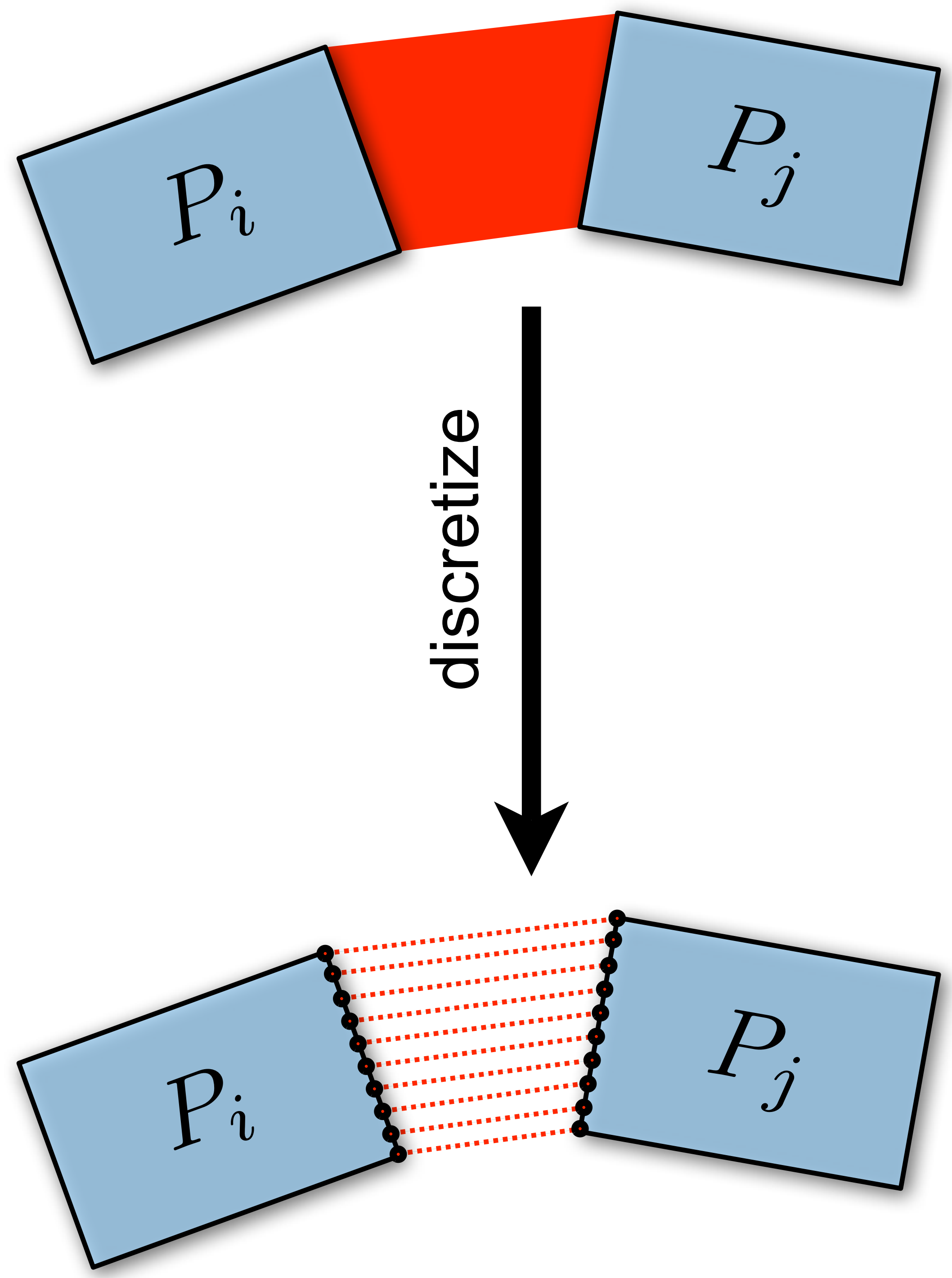
Physical Interpretation

$$E_{ij} = \int_{[0,1]^2} \left\| \mathbf{f}^{\,i\to j}(\mathbf{u}) - \mathbf{f}^{\,j\to i}(\mathbf{u}) \right\|^2 d\mathbf{u}$$

Integral over infinitesimal spring fibres

Sum of spring energies

$$E_{ij} \approx \sum_k \left\| \mathbf{f}_k^{i\to j} - \mathbf{f}_k^{j\to i} \right\|^2$$



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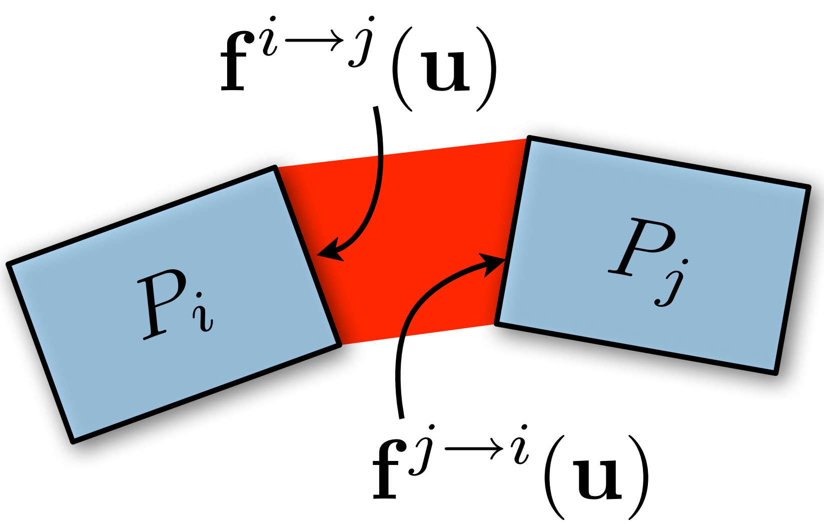
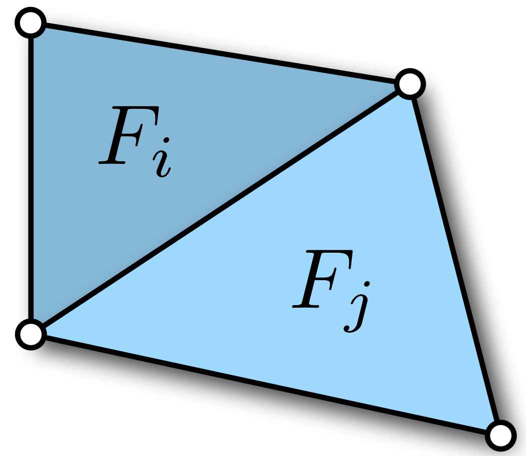


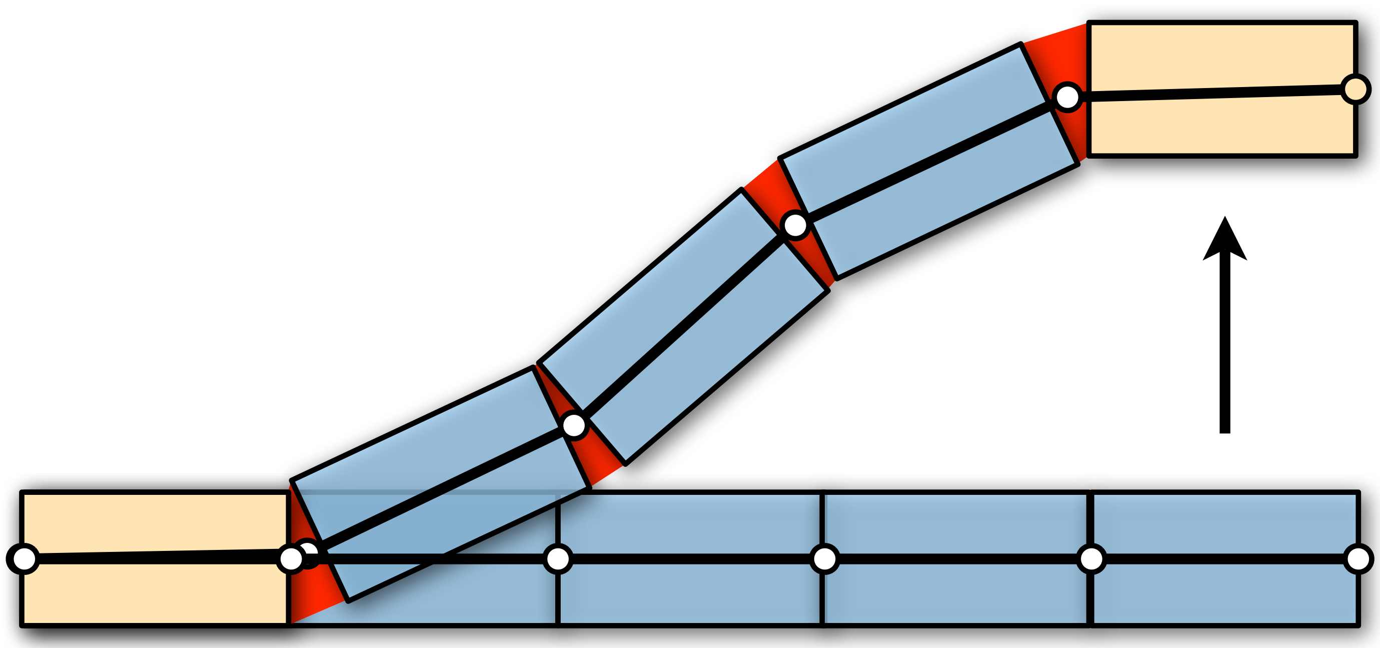
- Global energy

$$E = \sum_{\{i,j\}} w_{ij} \cdot E_{ij} \quad , \quad w_{ij} = \frac{\|\mathbf{e}_{ij}\|^2}{|F_i| + |F_j|}$$



Prism-Based Surface Deformation

1. Prescribes position/orientation for prisms
2. Find optimal rigid motions per prism
3. Update vertices by average prism transformations



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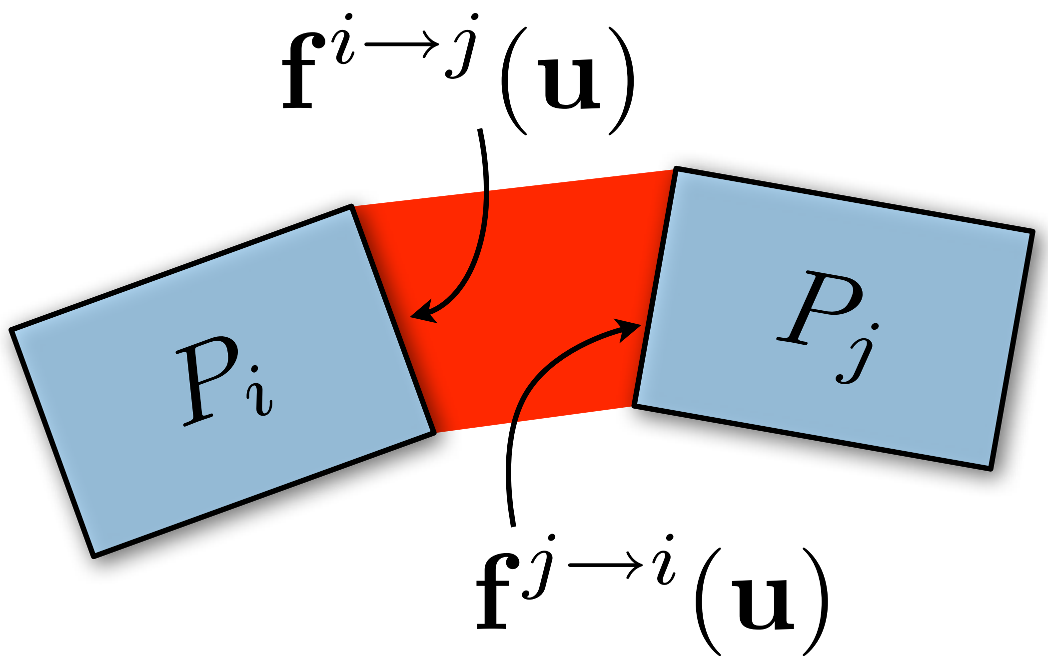
Non-Linear Minimization

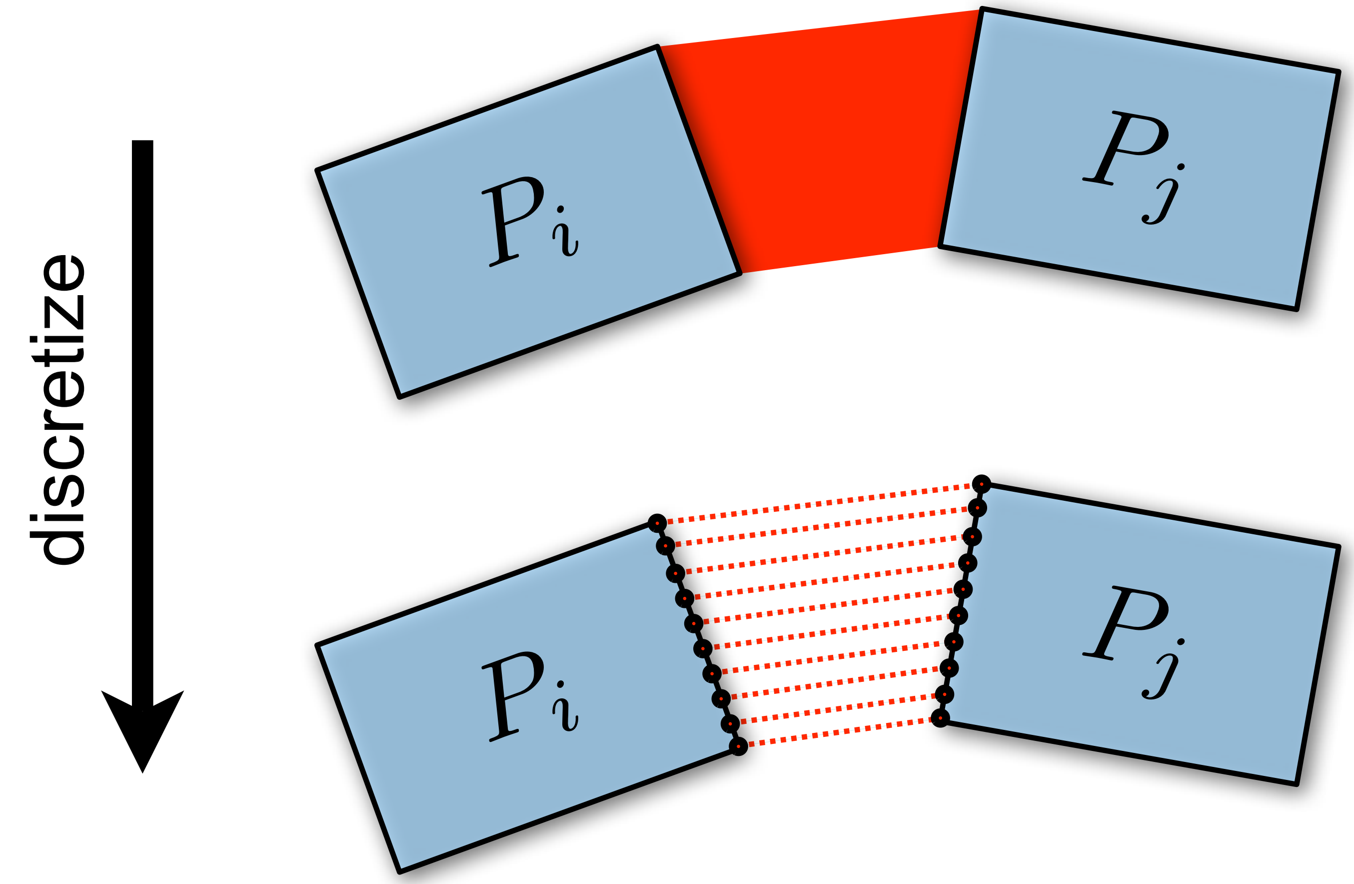
- Find rigid motion $(\mathbf{R}_i, \mathbf{t}_i)$ per prism P_i

$$\min_{\{\mathbf{R}_i,\,\mathbf{t}_i\}} \sum_{\{i,j\}} w_{ij} \int_{[0,1]^2} \left\| \mathbf{R}_i\,\mathbf{f}^{\,i\to j}(\mathbf{u}) + \mathbf{t}_i \;-\; \mathbf{R}_j\,\mathbf{f}^{\,j\to i}(\mathbf{u}) - \mathbf{t}_j \right\|^2 d\mathbf{u}$$



Continuous Shape Matching

$$\min_{\{\mathbf{R}_i,\,\mathbf{t}_i\}} \sum_{\{i,j\}} w_{ij} \int_{[0,1]^2} \left\| \mathbf{R}_i\, \mathbf{f}^{\,i\to j}(\mathbf{u}) + \mathbf{t}_i \;-\; \mathbf{R}_j\, \mathbf{f}^{\,j\to i}(\mathbf{u}) - \mathbf{t}_j \right\|^2 d\mathbf{u}$$



$$\min_{\{\mathbf{R}_i,\,\mathbf{t}_i\}} \sum_{\{i,j\}} w_{ij} \sum_{k} \left\| \mathbf{R}_i\, \mathbf{f}_k^{\,i\to j} + \mathbf{t}_i \;-\; \mathbf{R}_j\, \mathbf{f}_k^{\,j\to i} - \mathbf{t}_j \right\|^2 d\mathbf{u}$$

Non-Linear Minimization

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- Generalized shape matching problem
  – Discrete point correspondences vs. continuous face correspondences

➡ Adapt techniques for point-set registration

Iterated Local Shape Matching

- Iterate this:
  - Randomly pick one prism
  - Optimize its position/orientation [Horn87]

$$\min_{\mathbf{R}_i\,,\,\mathbf{t}_i} \sum_{j \in \mathcal{N}_i} w_{ij} \int_{[0,1]^2} \left\| \mathbf{R}_i\, \mathbf{f}^{\,i \to j}(\mathbf{u}) + \mathbf{t}_i \;-\; \mathbf{f}^{\,j \to i}(\mathbf{u}) \right\|^2 d\mathbf{u}$$

- Corresponds to error diffusion
  - Rapidly removes high error frequencies
  - Impractically slow convergence

Global Shape Matching [Pottmann 04]

- First order approx. of rigid motions

$$\mathbf{R}_i\,(\cdot) + \mathbf{t}_i \;\approx\; (\cdot) \;+\; \omega_i \times (\cdot) \;+\; \mathbf{v}_i \;=:\; \mathbf{A}_i\,(\cdot)$$

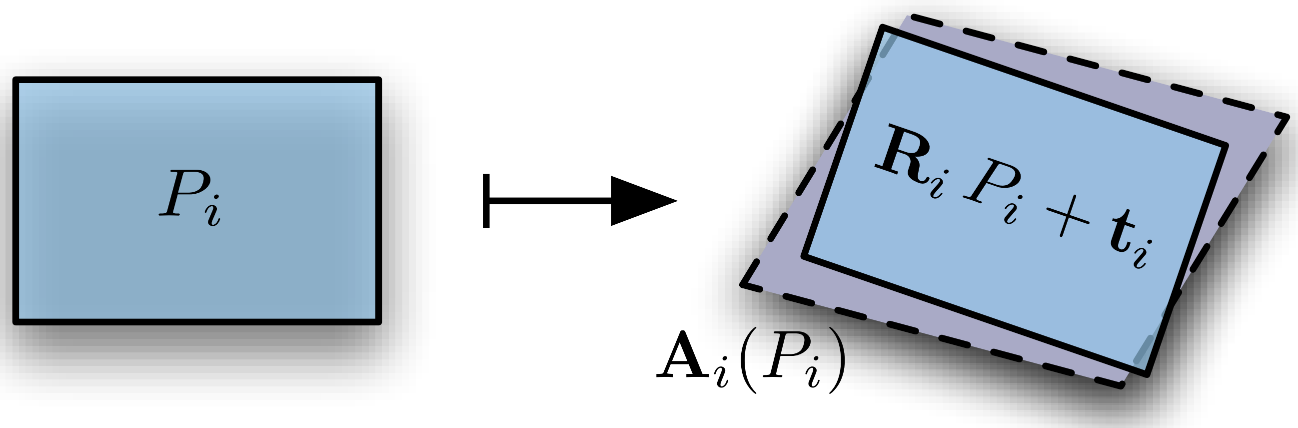
- Quadratic minimization wrt. velocities

$$\min_{\{\mathbf{v}_i\,,\,\omega_i\}} \sum_{\{i,j\}} w_{ij} \int_{[0,1]^2} \left\| \mathbf{A}_i \left(\mathbf{f}^{\,i \to j}(\mathbf{u}) \right) - \mathbf{A}_j \left(\mathbf{f}^{\,j \to i}(\mathbf{u}) \right) \right\|^2 d\mathbf{u}$$

- Yields affine motion $\mathbf{A}_i$ per prism
  - Project to manifold of rigid motions

Global Shape Matching

- Find "closest" rigid motion
  - Measure distance of transformations' images
  - Another local shape matching



- Larger steps, fewer iterations
  - Factor 50 faster than [Pottmann02]

Global Shape Matching

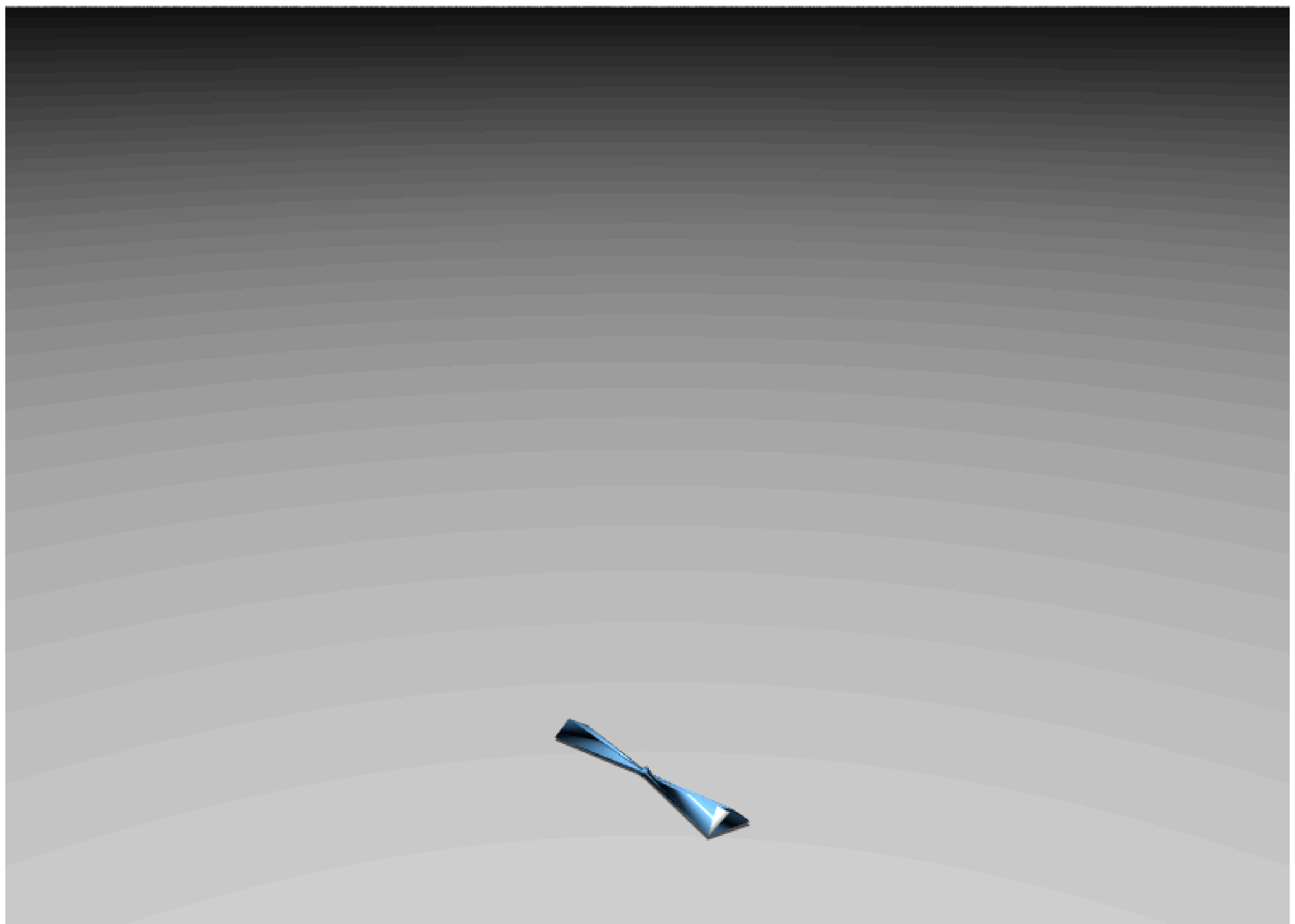
```
while not converged
{
   find optimal velocities [vi,wi]
   ∀i: (Ri,ti) = project(vi,wi)
   ∀i: Pi = Ri*Pi + ti
}
```

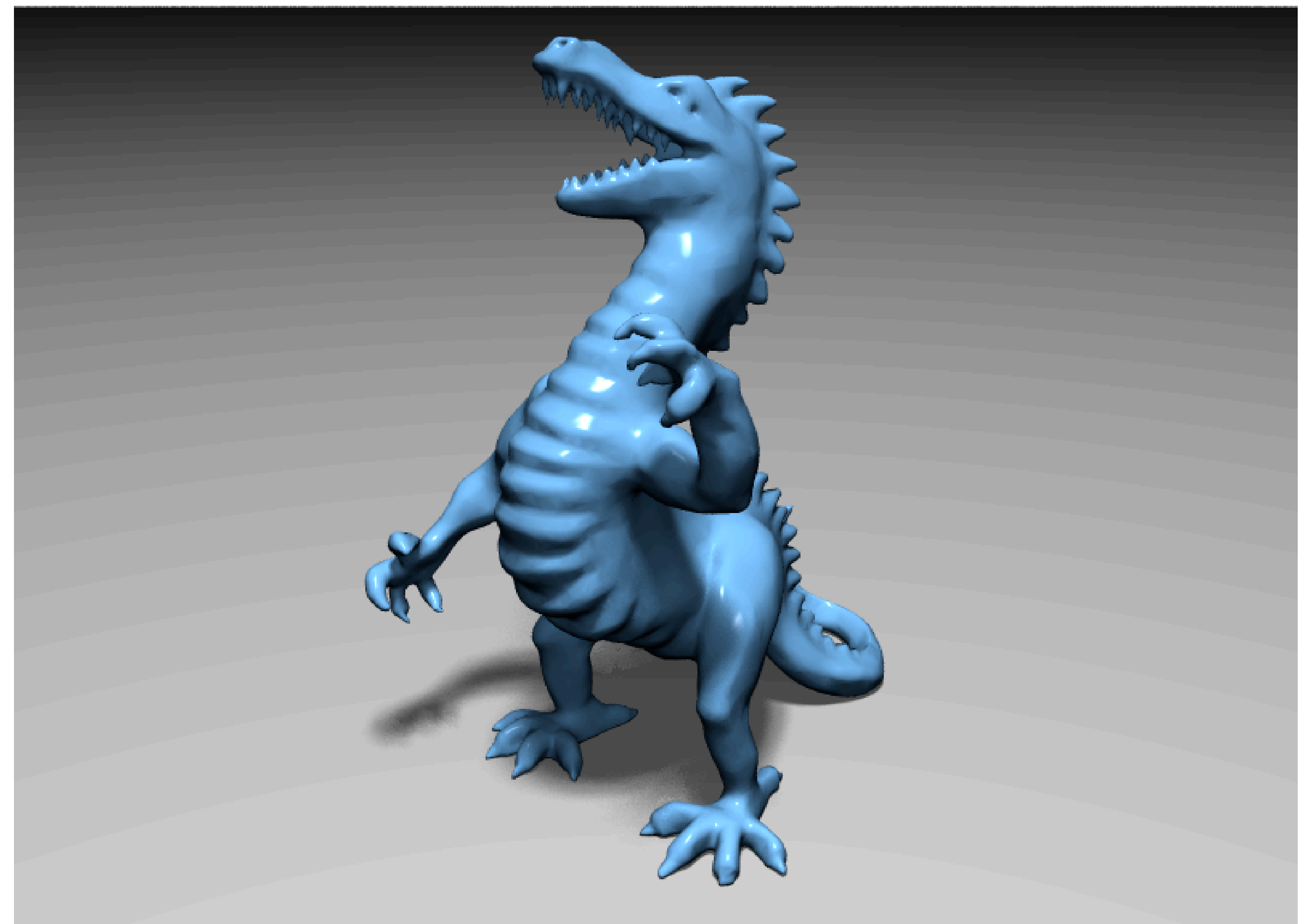
Performance: ~7k prism updates per second

Hierarchical Shape Matching

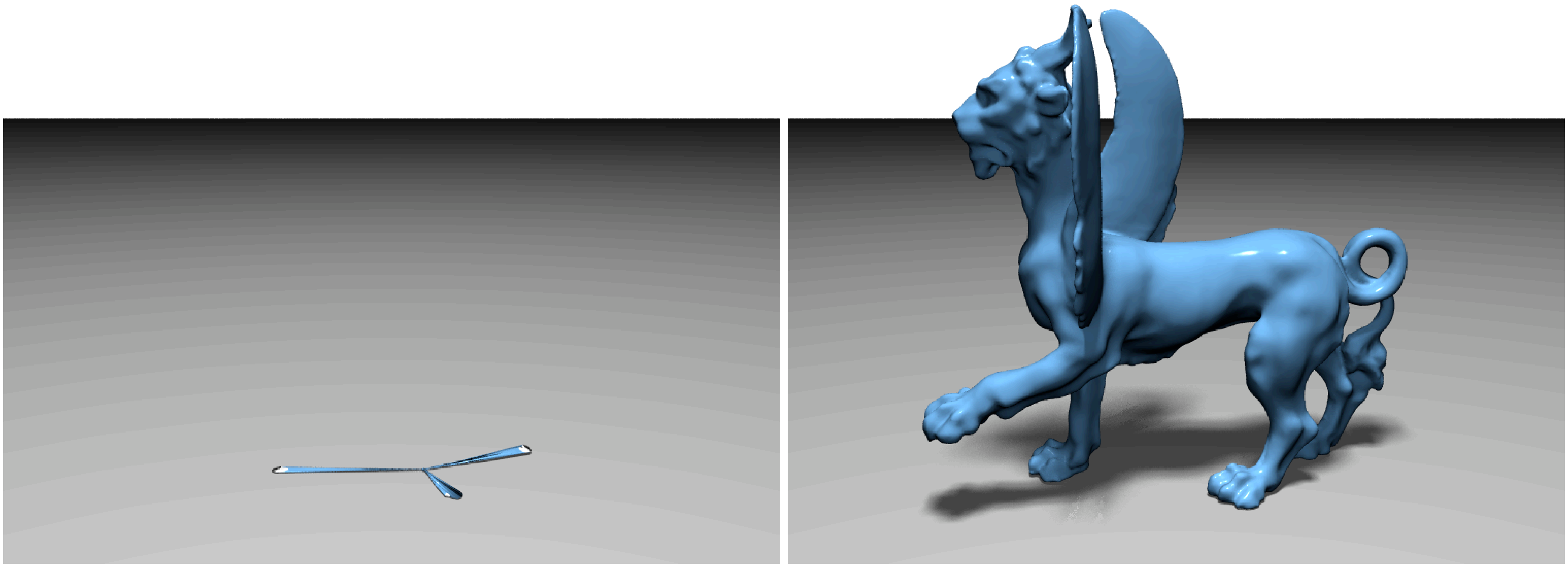
- Local and global matching alone don't work
  - Slow convergence of local matching
  - High complexity of global matching
- Hierarchical multi-grid matching
  - Solve global matching on coarse level
  - Apply local matching on finer levels

Robustness





Robustness



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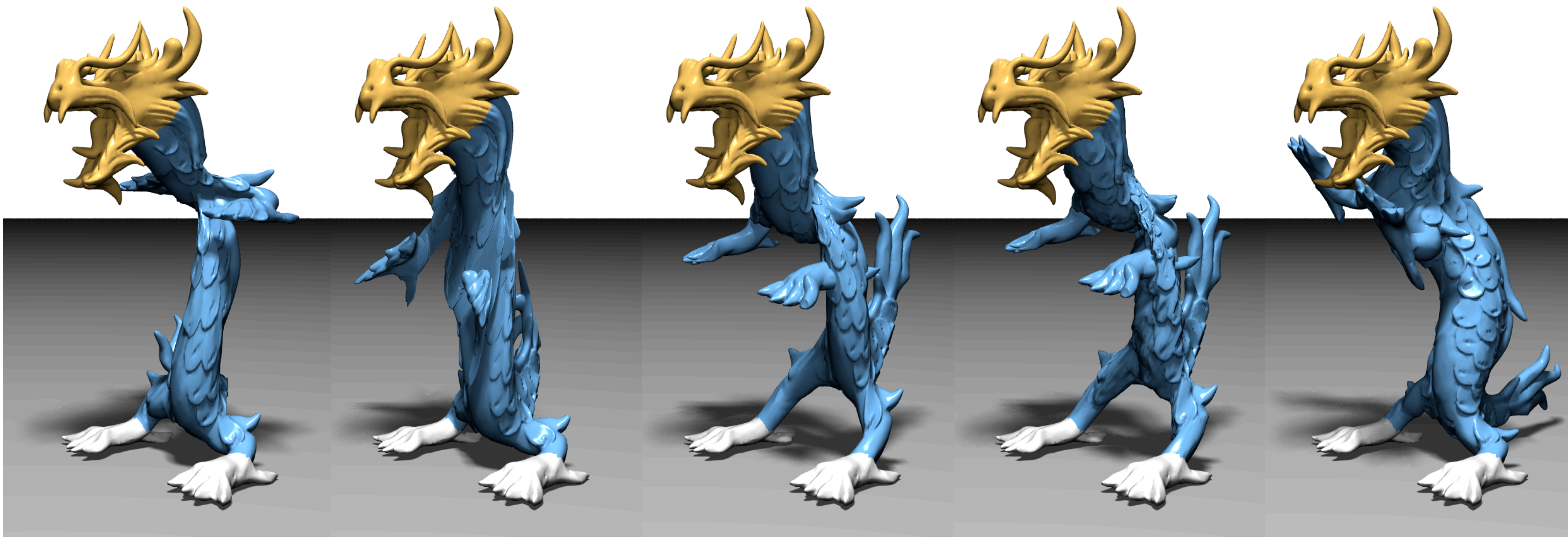
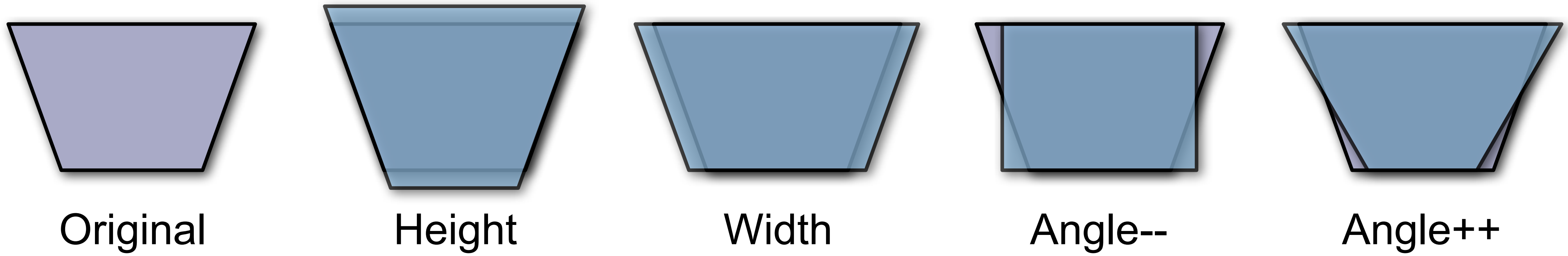
- **Results**

Dragon Deformation

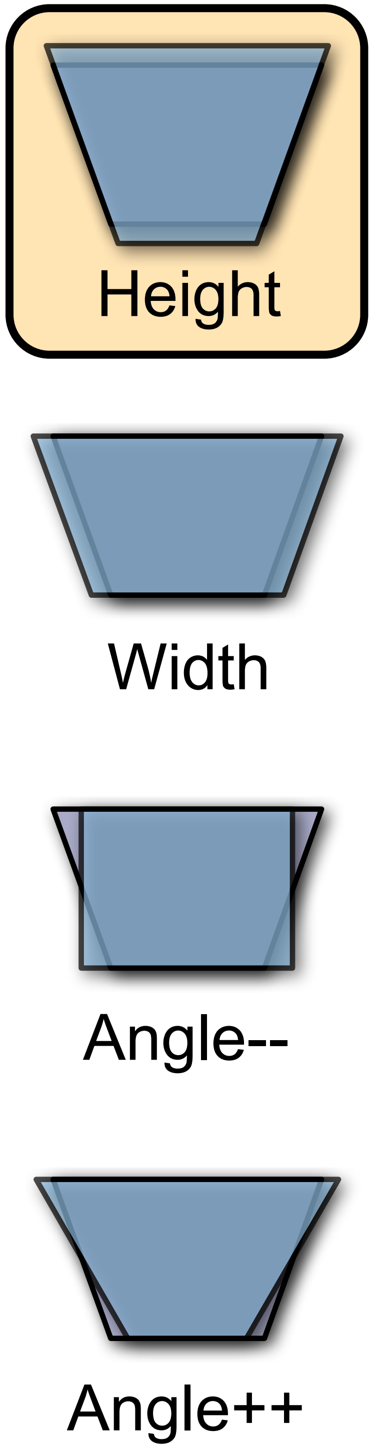


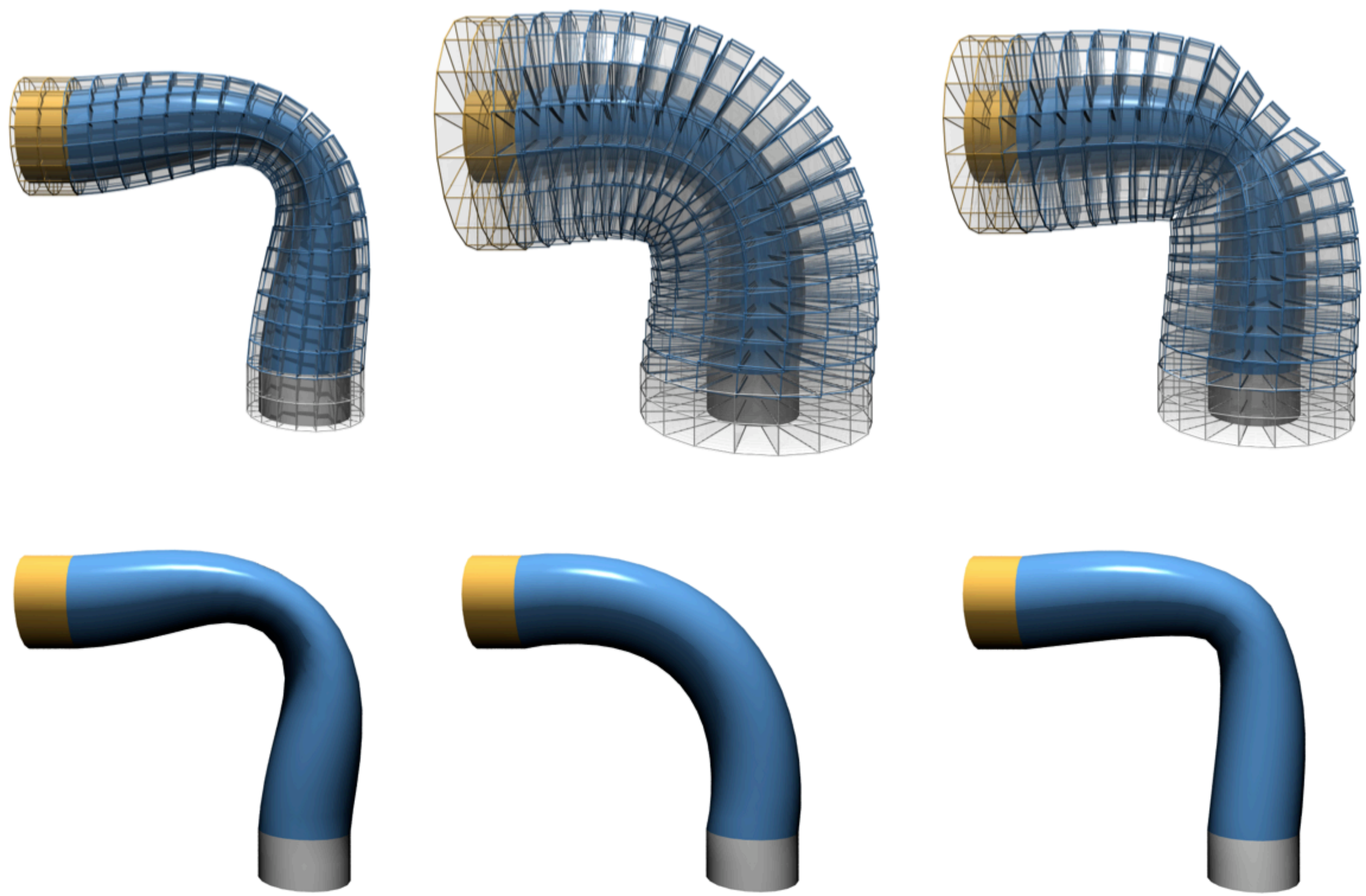
VarMin RBF Grad RotInv PriMo

Prism Parameters

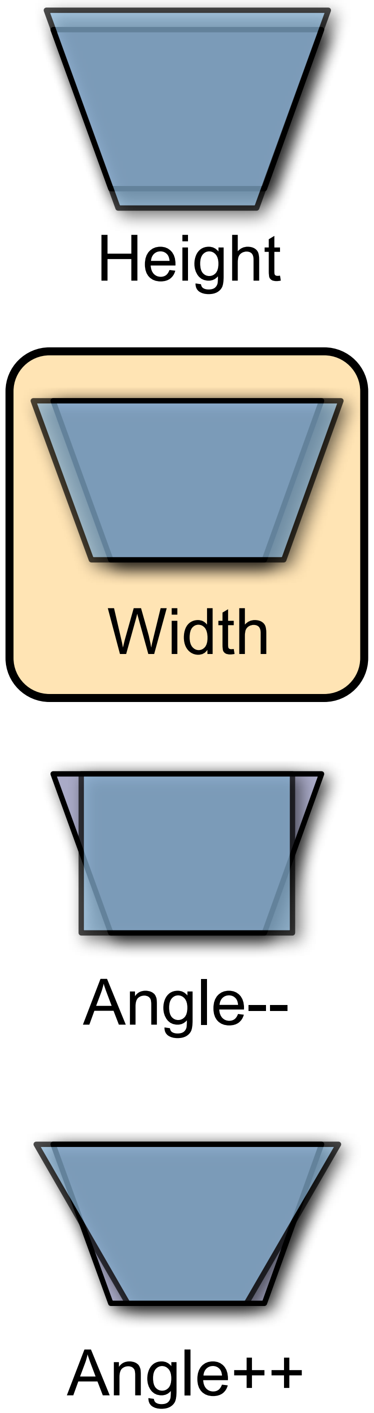


Stiffness Control

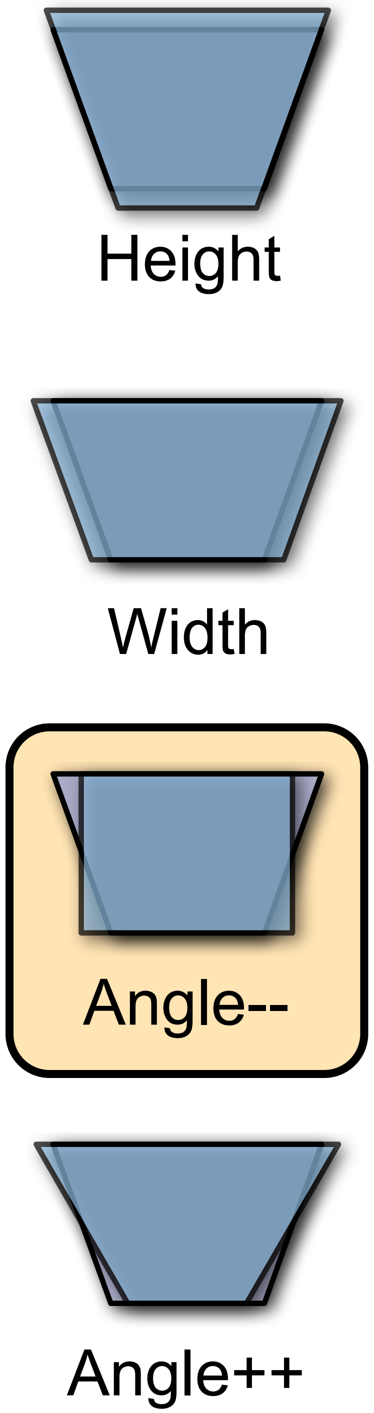


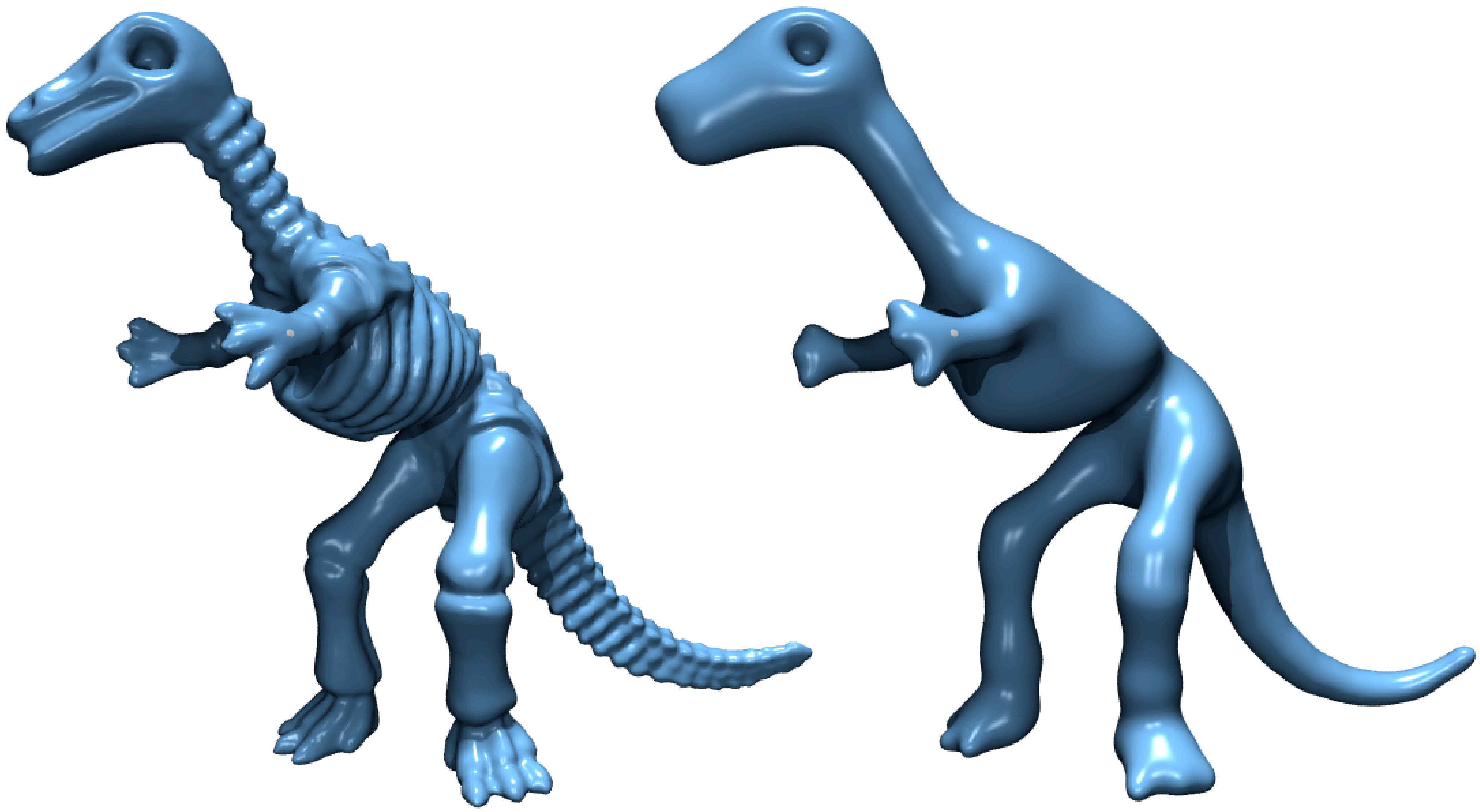


Control Surface Area

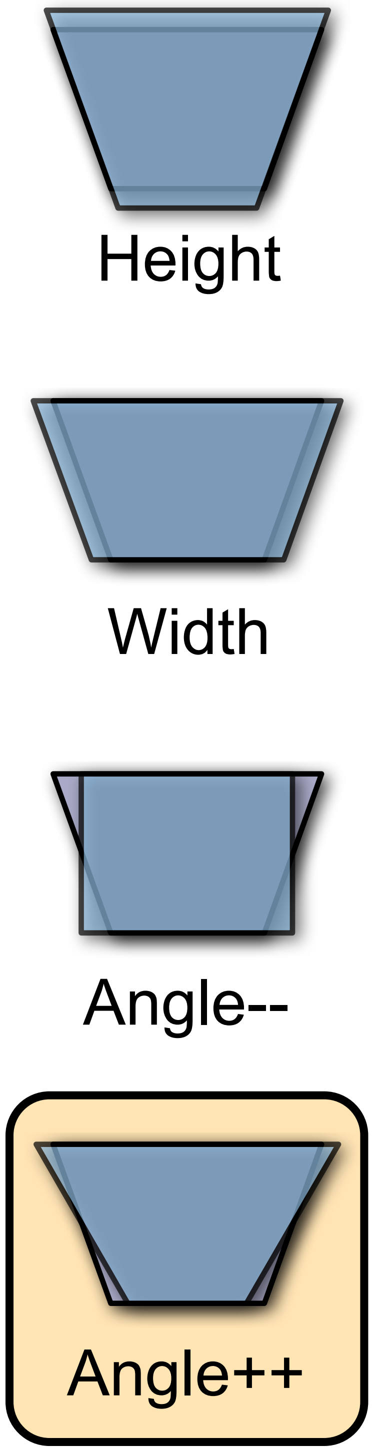


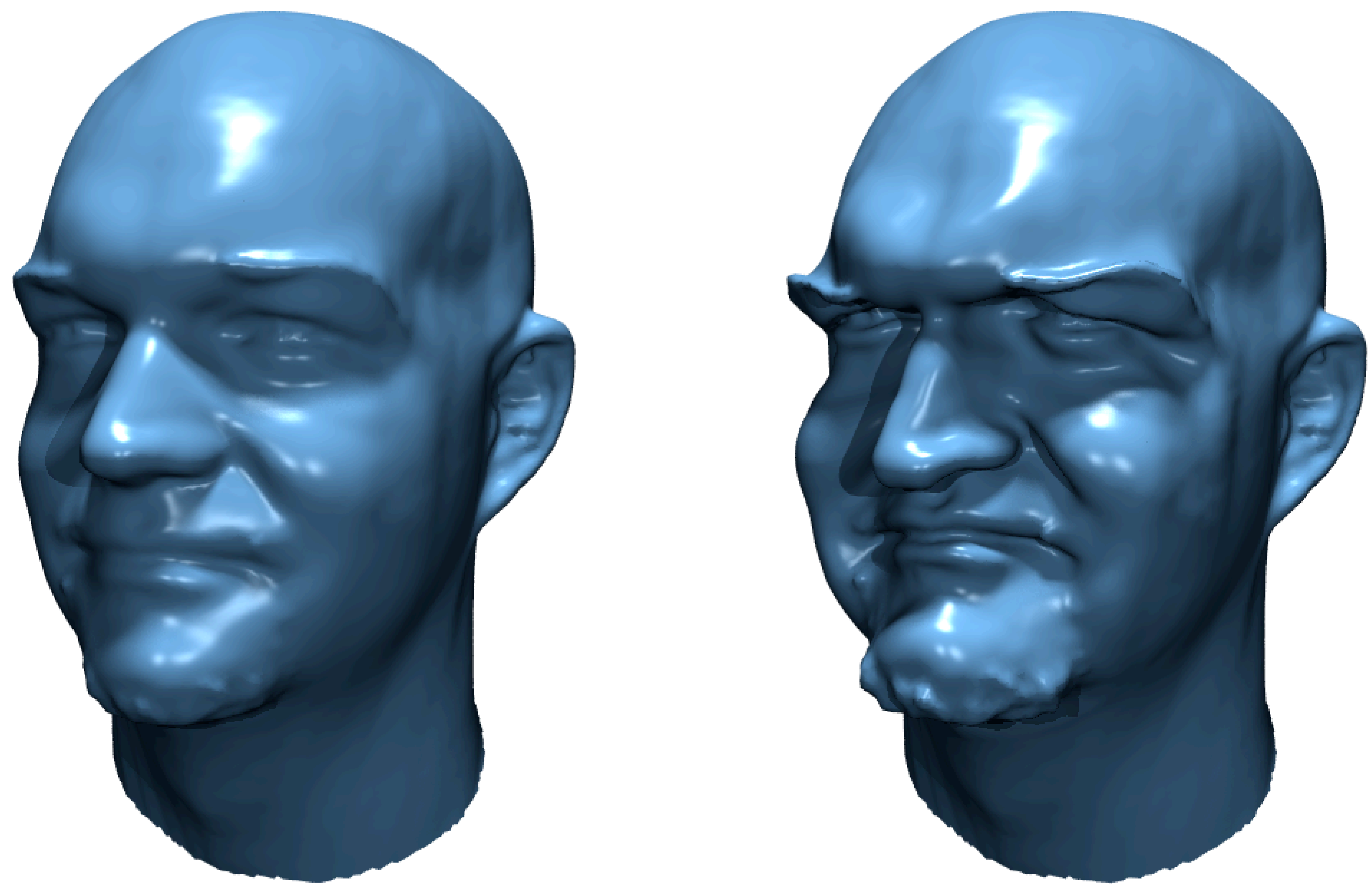
Non-Shrinking Smoothing





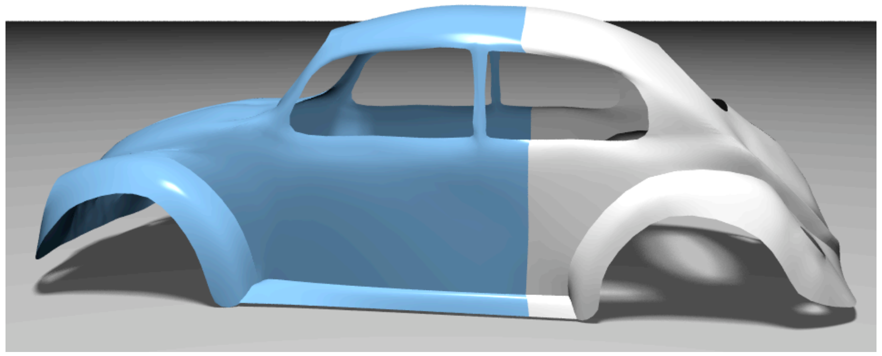
Detail Enhancement

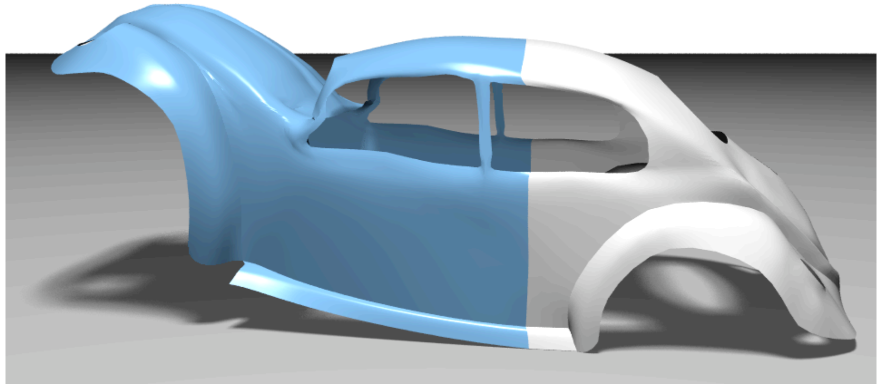


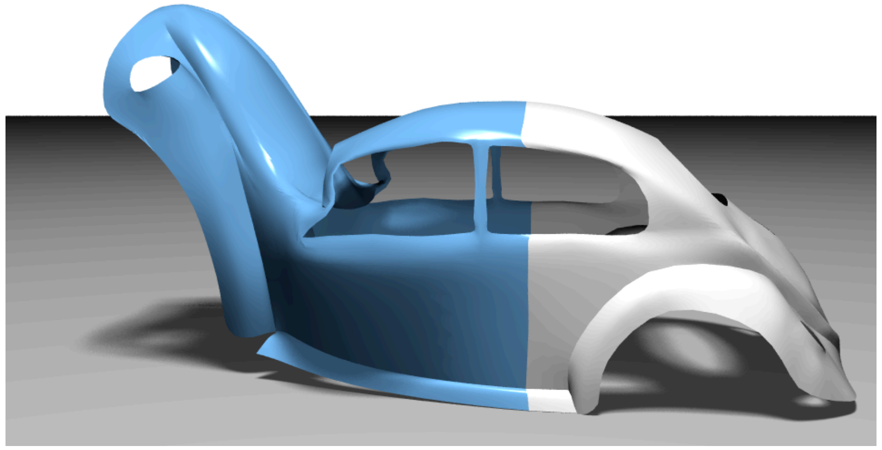


Force-Based Deformation

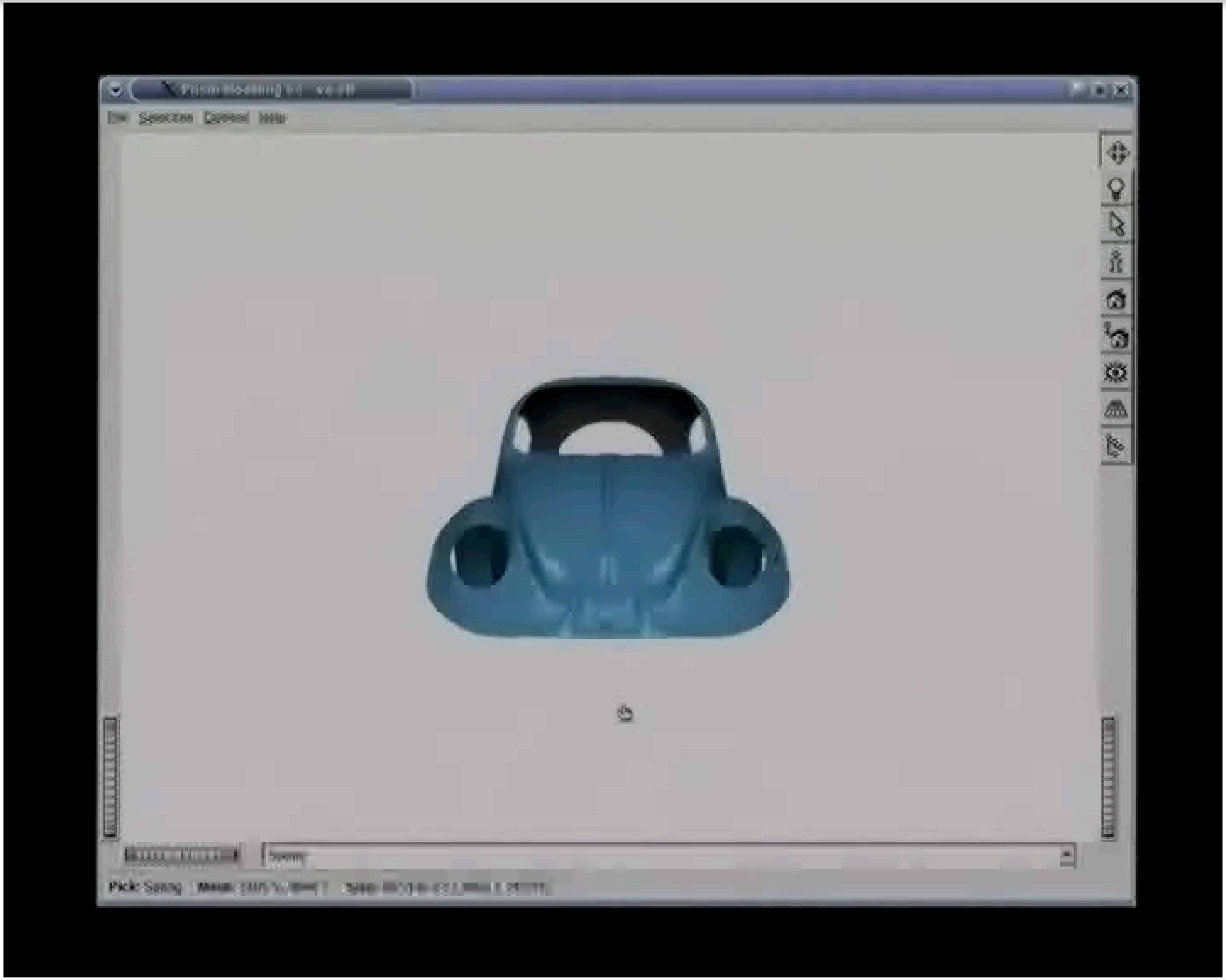
- Separately prescribe
  - positions and/or
  - orientations
- Forces can be more intuitive
  - Physically intuitive
  - Constraints = high forces
- Incorporate forces
  - Just another spring energy



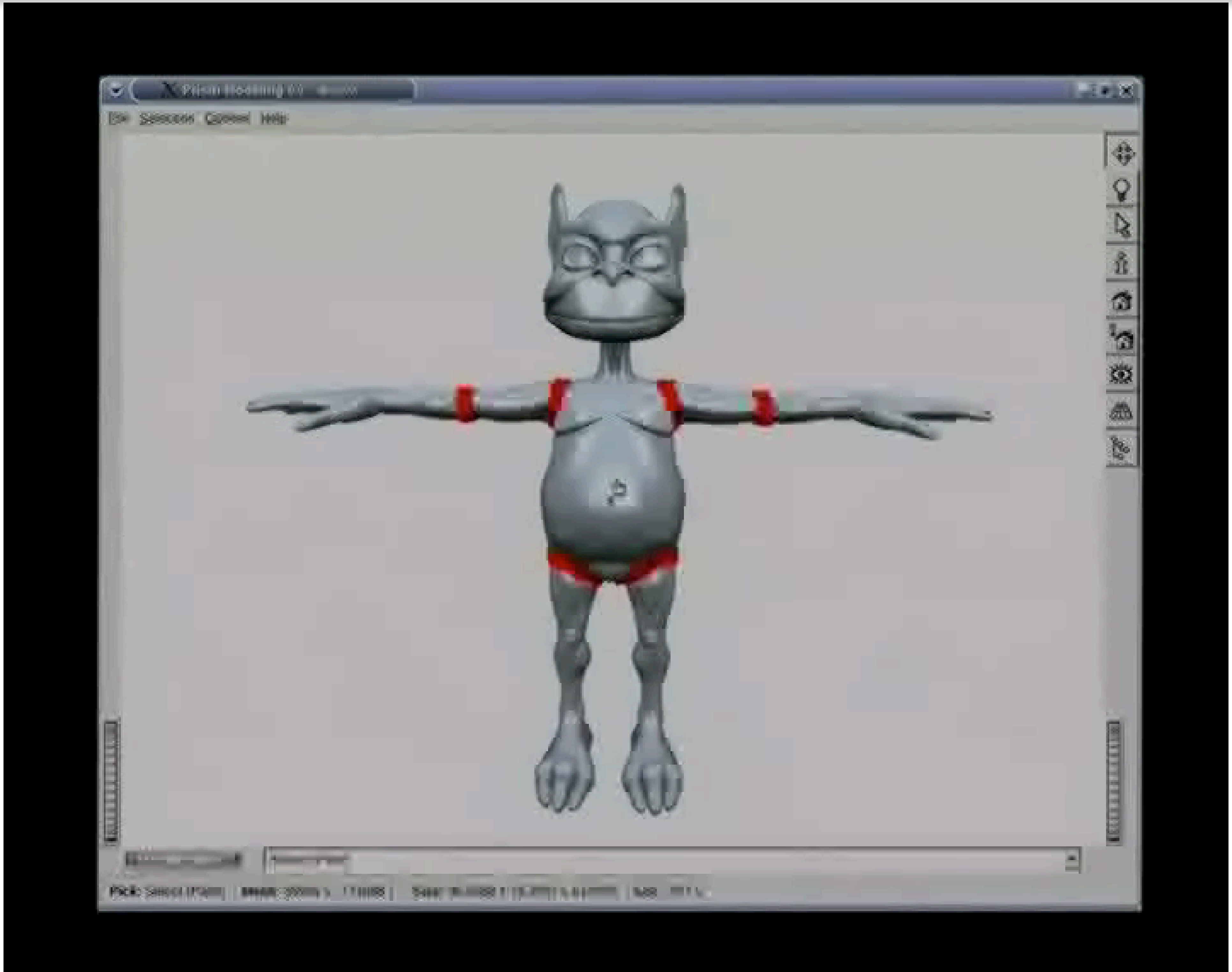




Force-Based Deformation

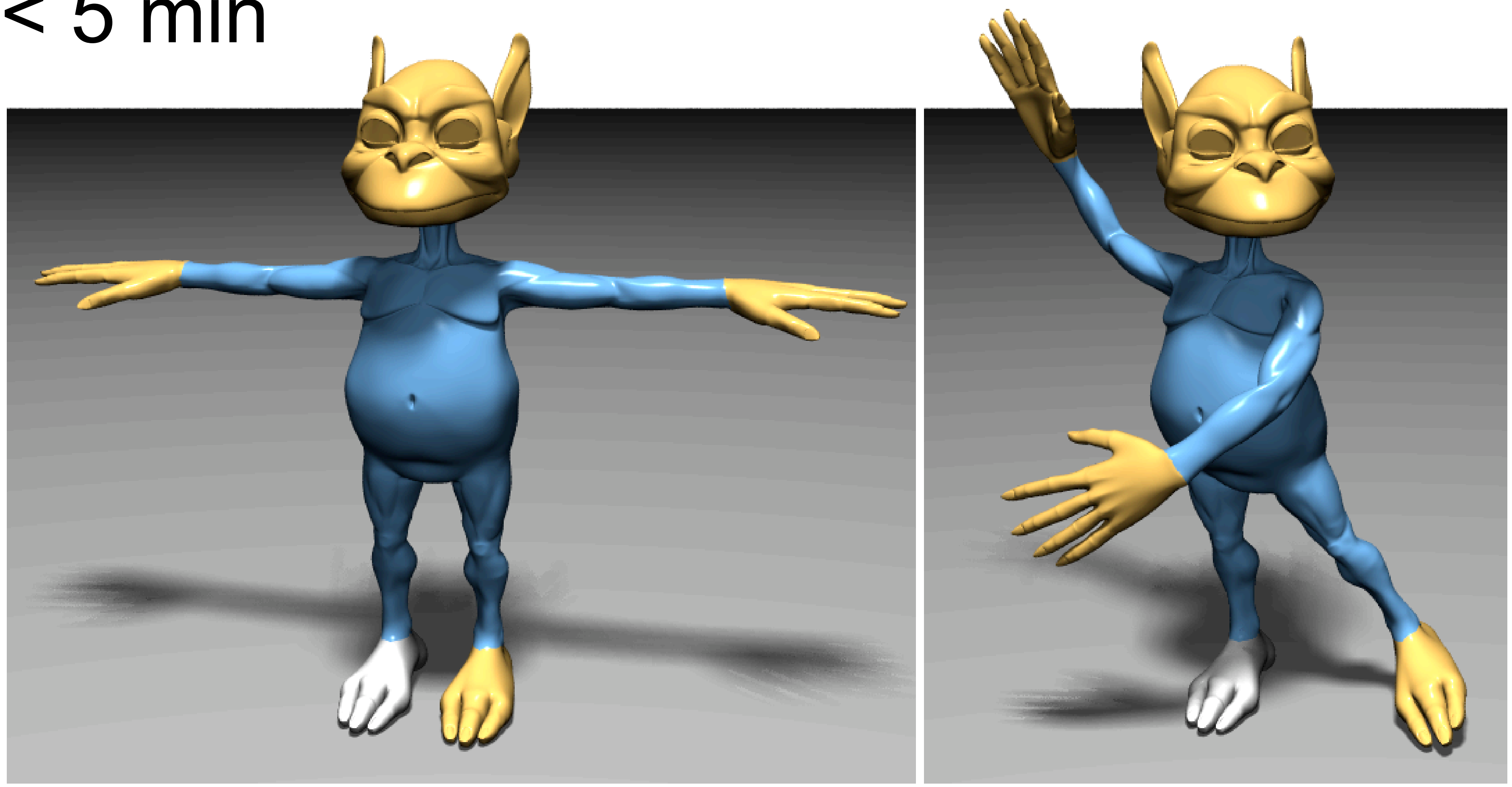


Goblin Posing



Goblin Posing

- Decreased stiffness at joints
- Forces & hard constraints
- 180k triangles at about 1 fps
- Whole session < 5 min



Conclusion

- Non-linear surface deformation model
  - Physically plausible
  - Intuitive parameters for surface behavior
  - Constraint-based and force-based
- Hierarchical shape matching
  - Extremely robust
  - Reasonably efficient
  - Easy implementation