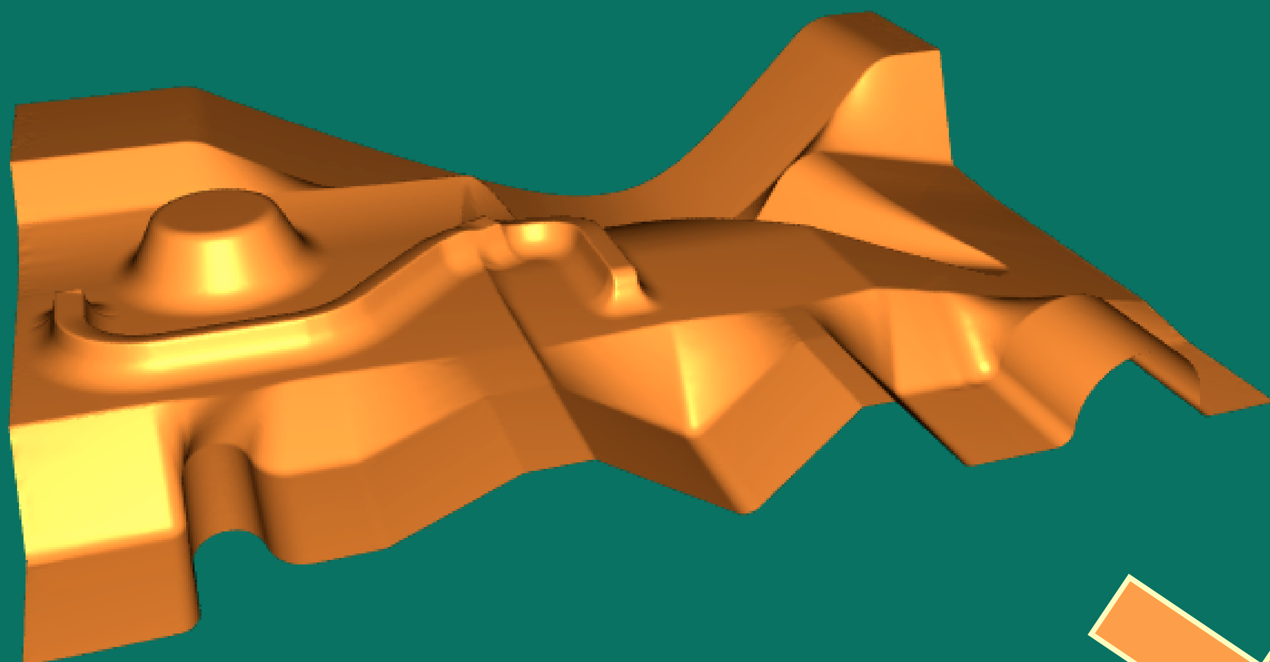
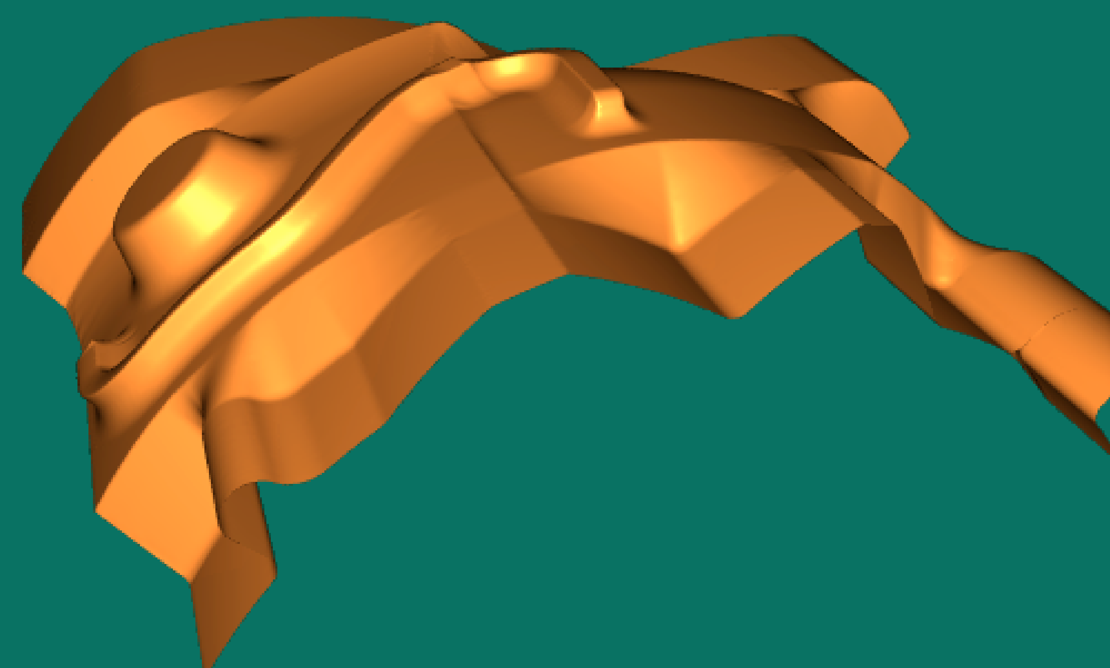
An Intuitive Framework for Real-Time Freeform Modeling

Mario Botsch Leif Kobbelt

Shape Deformation



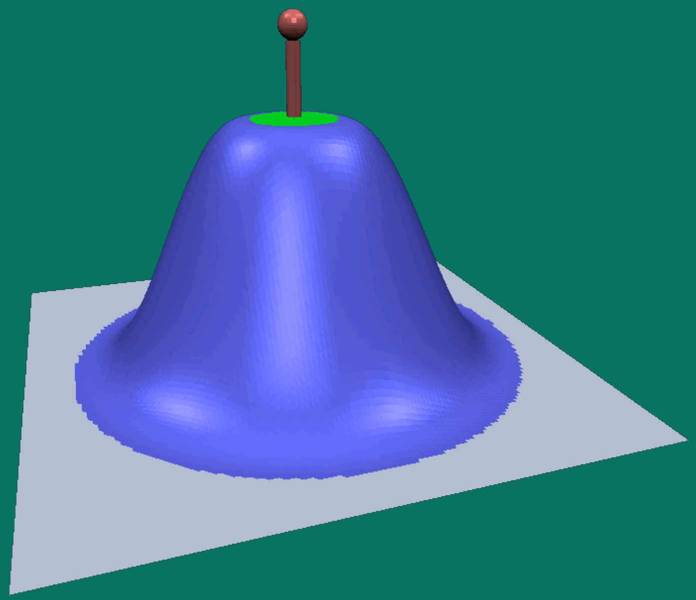
Complex shapes
Complex deformations



User Interaction

- Very limited user interface
  - 2D screen & mouse

- Intuitive metaphor needed
  - Control points / handles



Shape Deformation

- Move control handle to edit surface

$$S' = S + B\left(\delta C\right)$$

- Complex modification $B\left(\delta C\right)$?
  - Either complex user interaction δC
  - Or complex basis functions B

Shape Deformation

- Keep user interaction simple
  - Non-expert users
  - Limited user interface
- Need custom-tailored basis functions B
  - Arbitrary support
  - Smoothness
  - Surface stiffness & bending

Overview

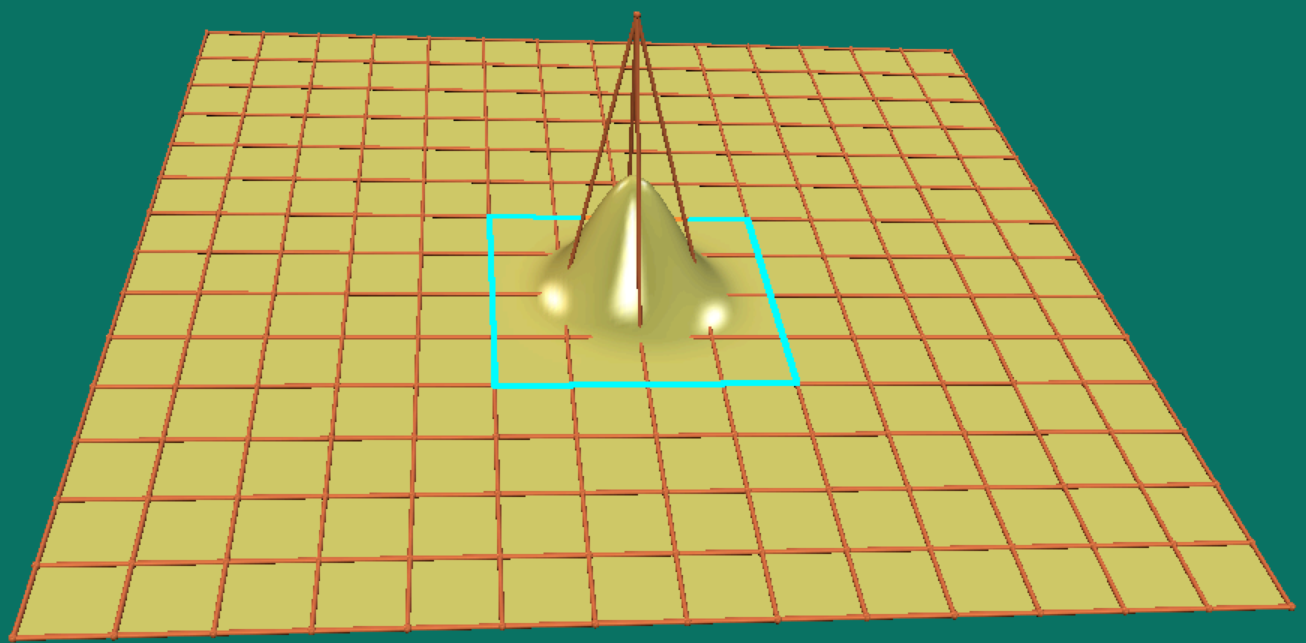
- Introduction
- Related Work
- Boundary constraint modeling
- Results
- Conclusion

Related Work

- NURBS & Subdivision surfaces
- Freeform Deformation
- Distance-Based Deformation
- Boundary Constraint Modeling

NURBS & Subdivision

- Basis functions are smooth bumps
  - Simple B, complex δC
  - Fixed support



NURBS & Subdivision

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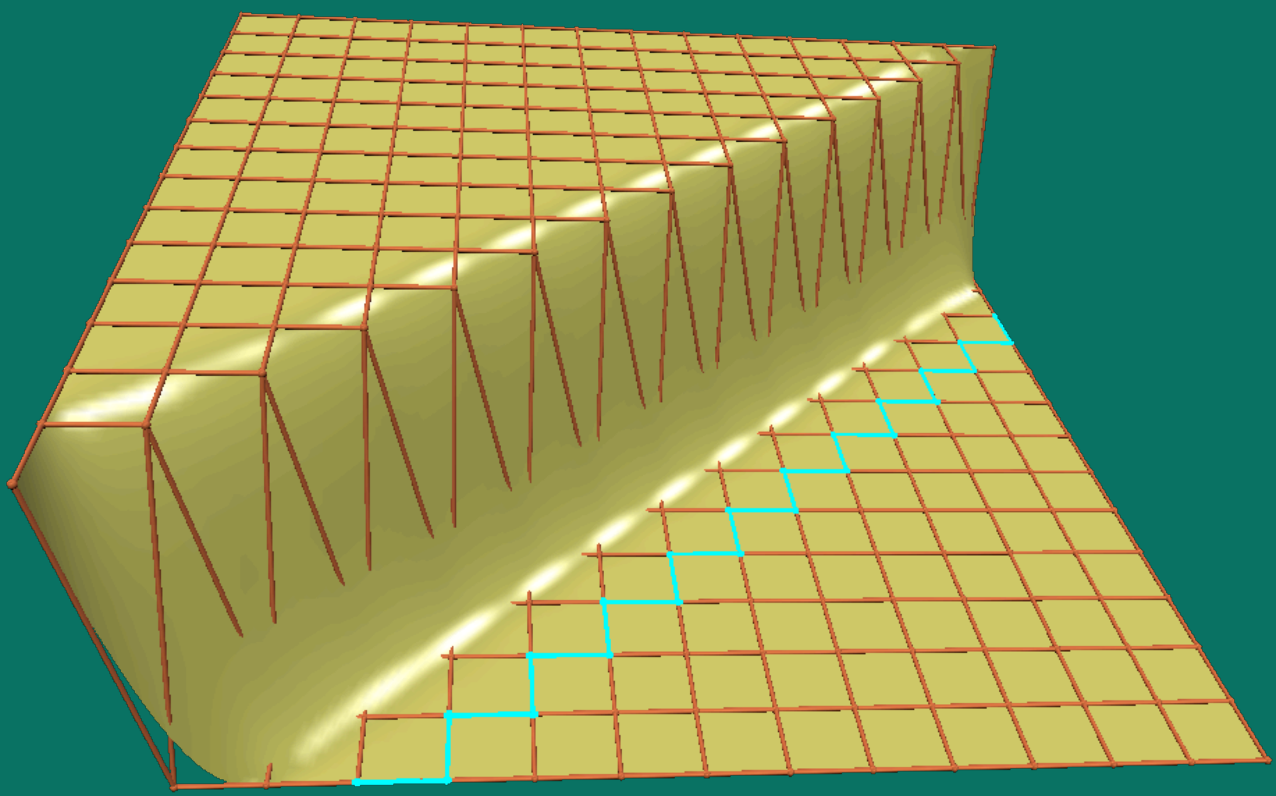

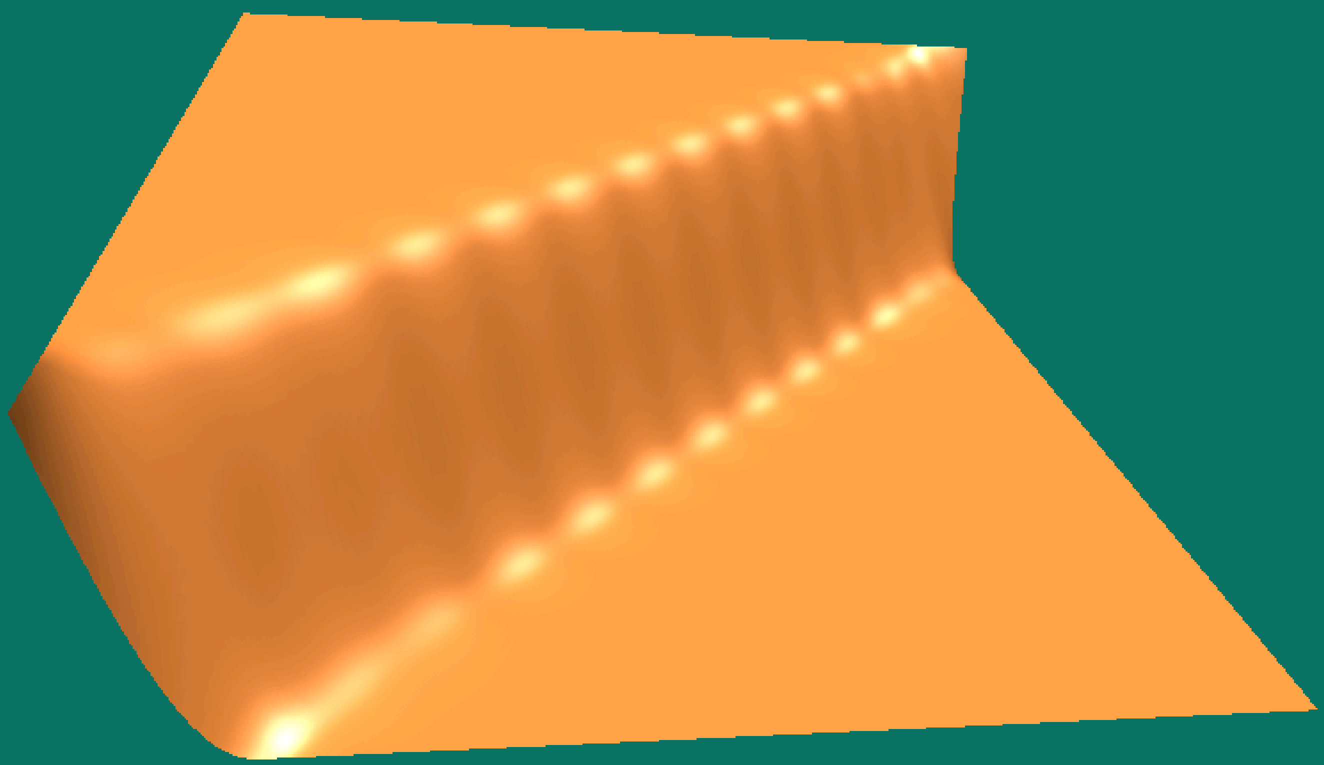

NURBS & Subdivision

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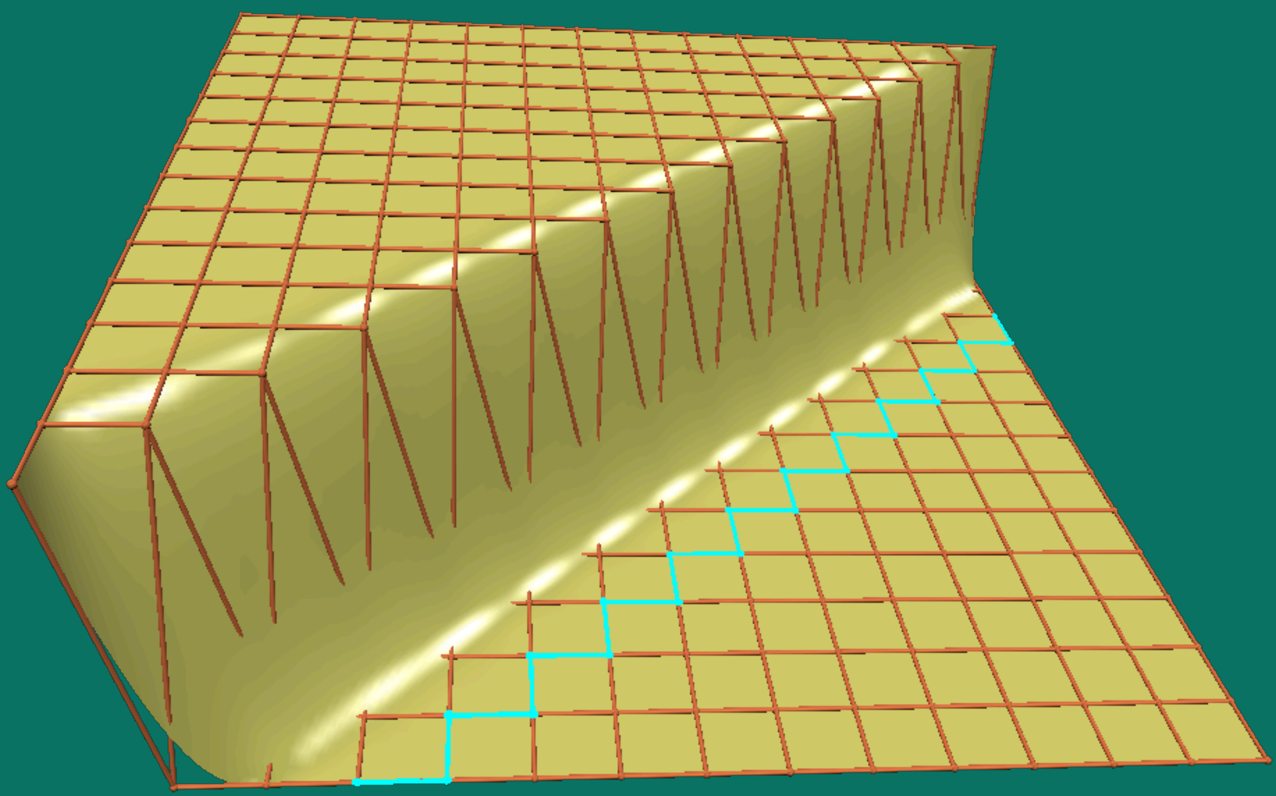
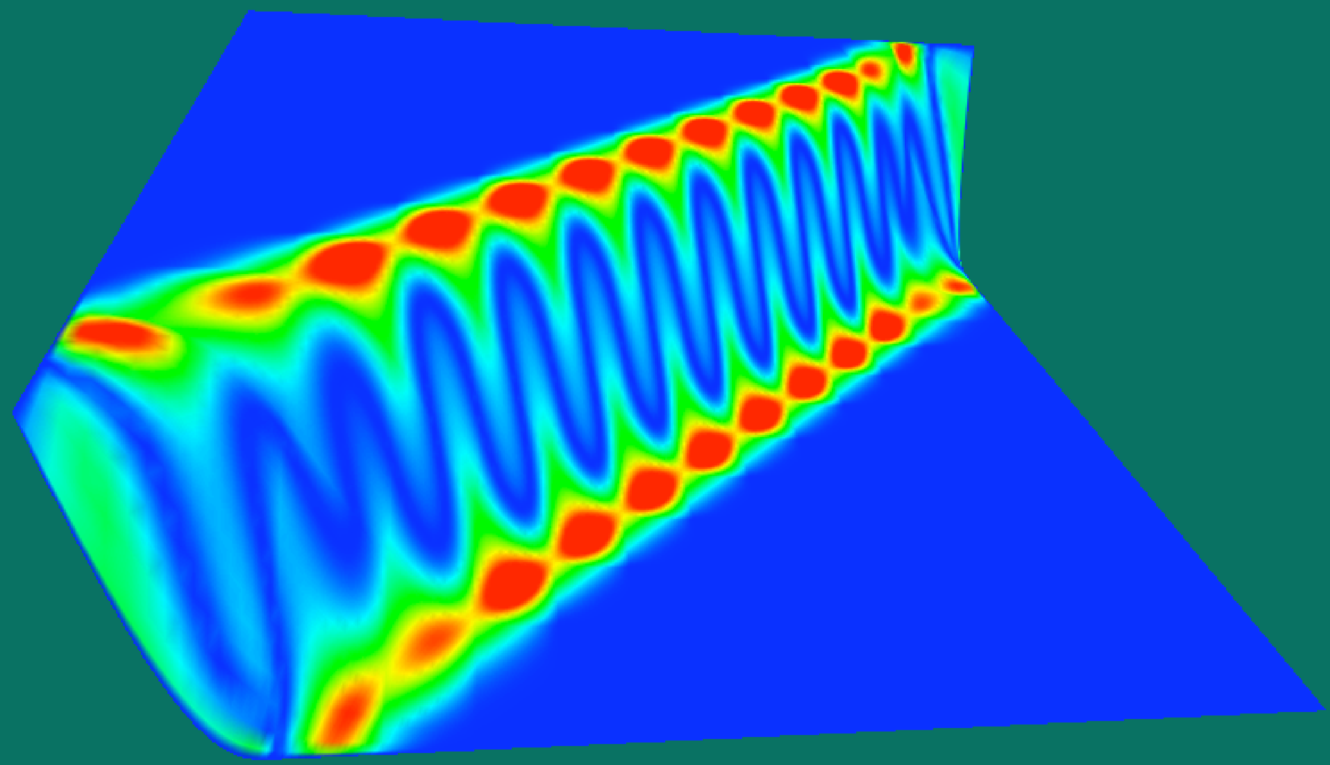
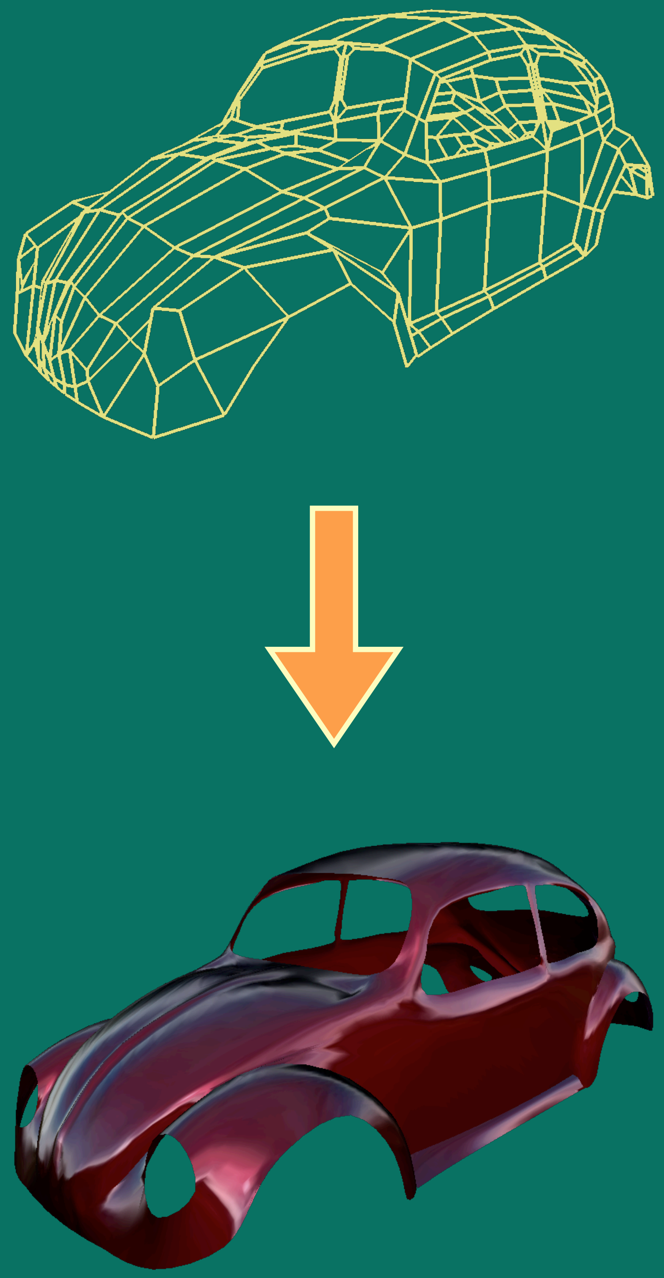




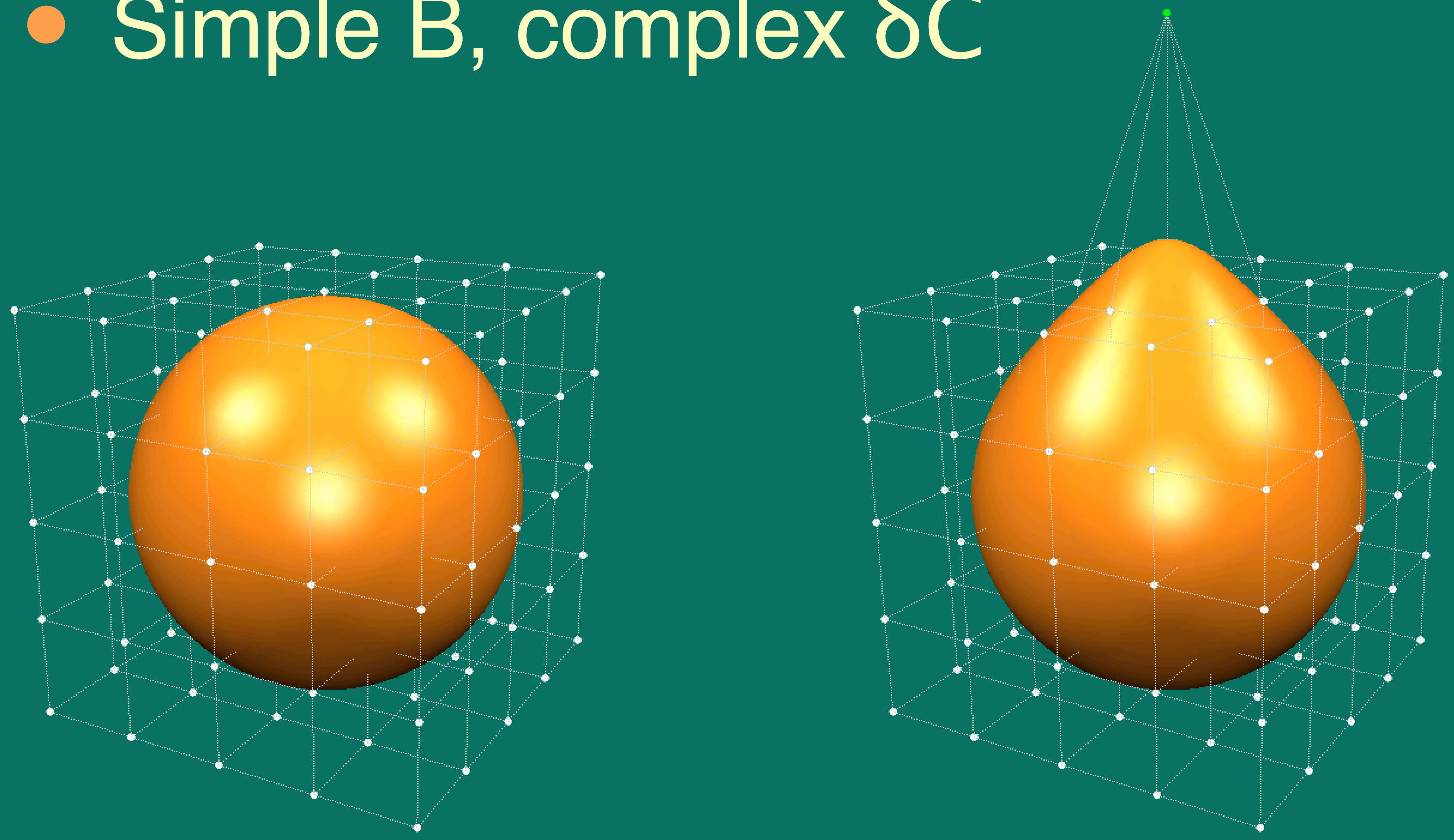
NURBS & Subdivision

- Fixed basis functions
  - Support
  - Number & position
- Bound to control points
  - Initial patch layout crucial
  - Requires experts!
- De-couple deformation basis from surface representations!



Freeform Deformation

- Deform space around object
  - Tri-variate tensor-product spline
  - Simple B, complex δC



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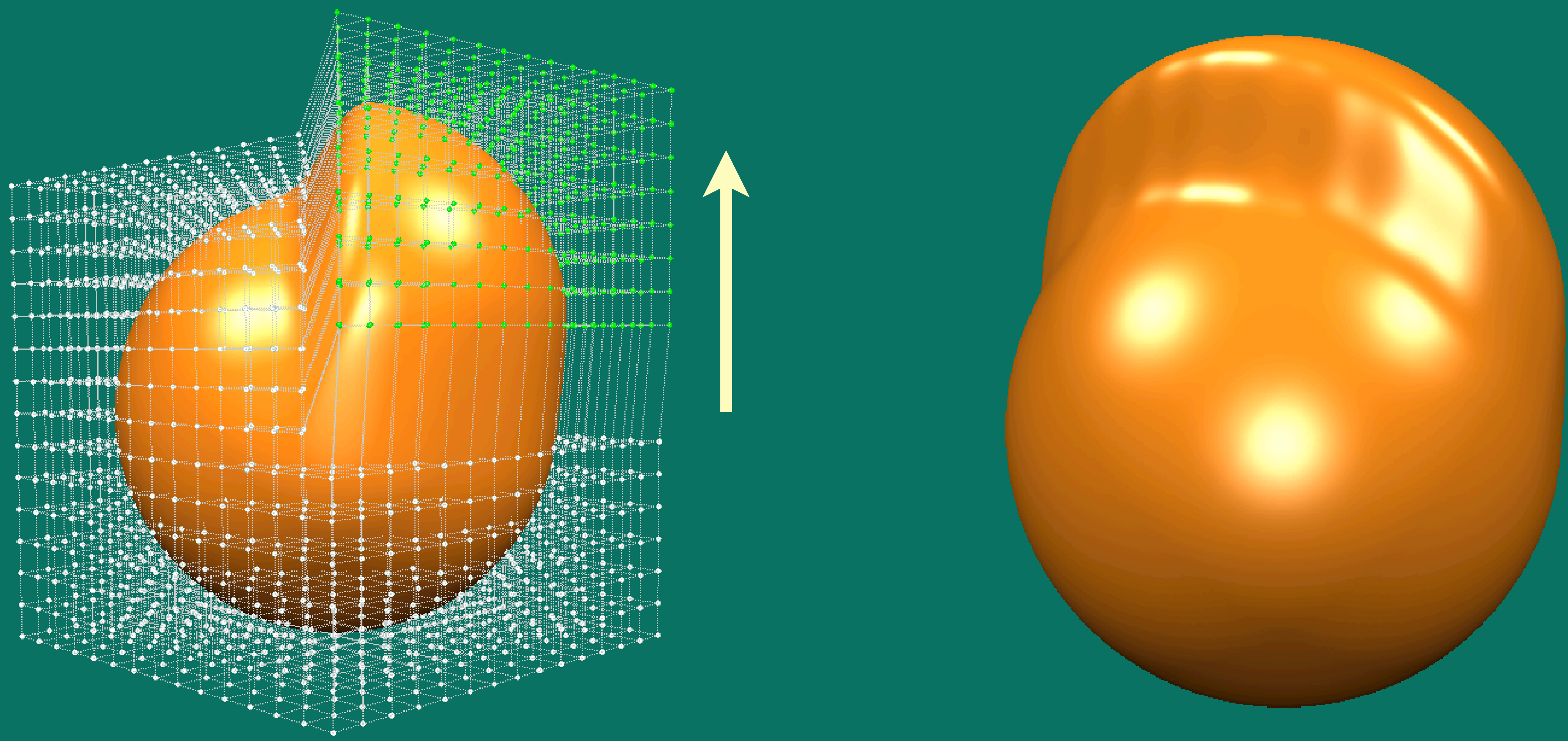

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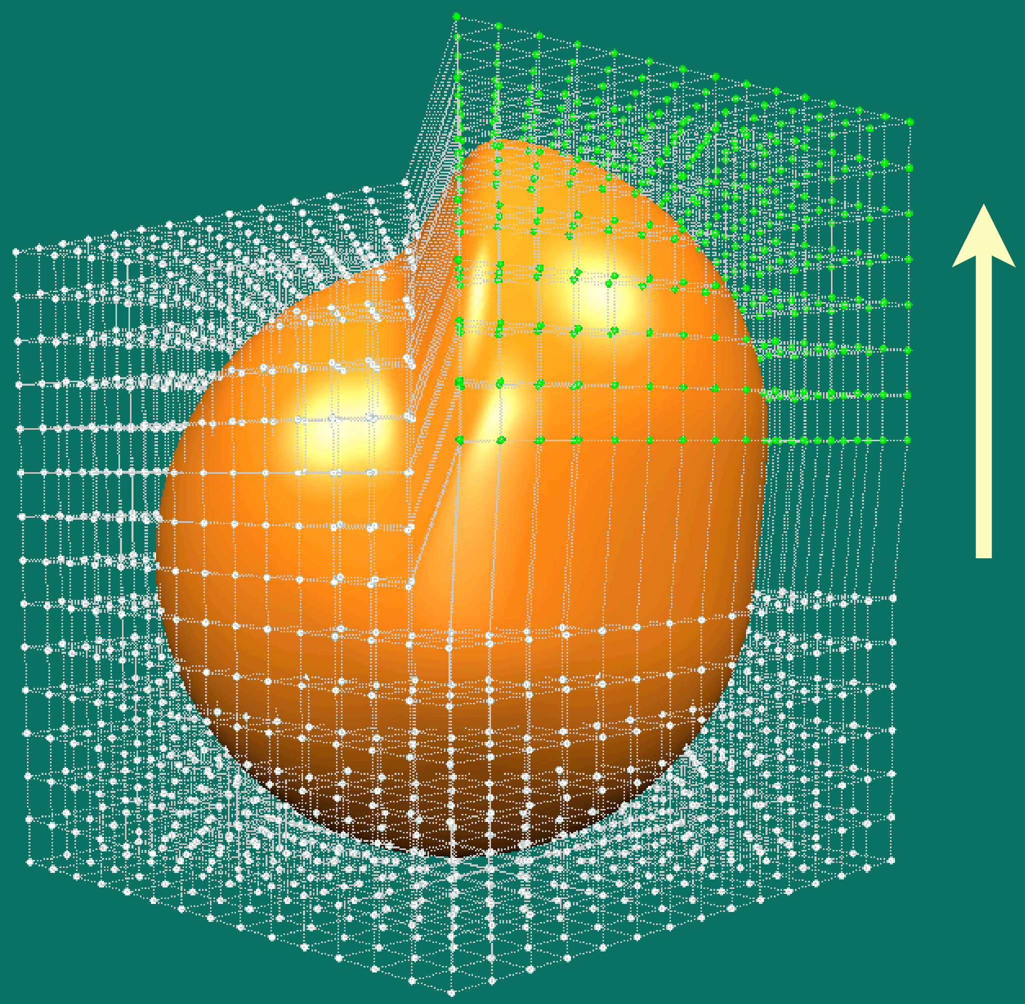
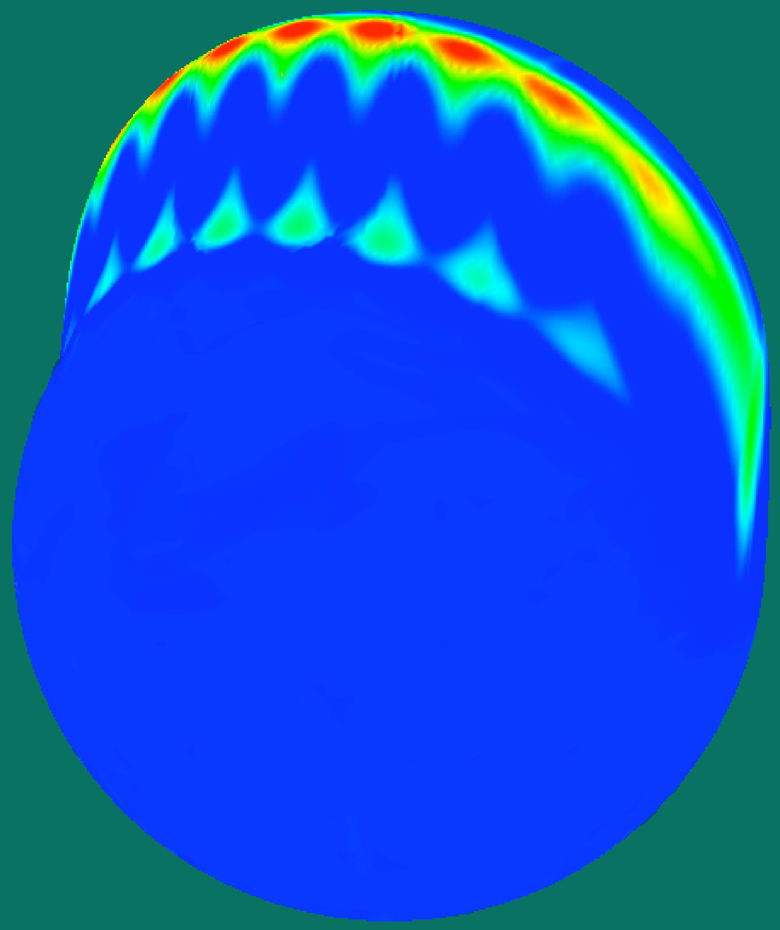
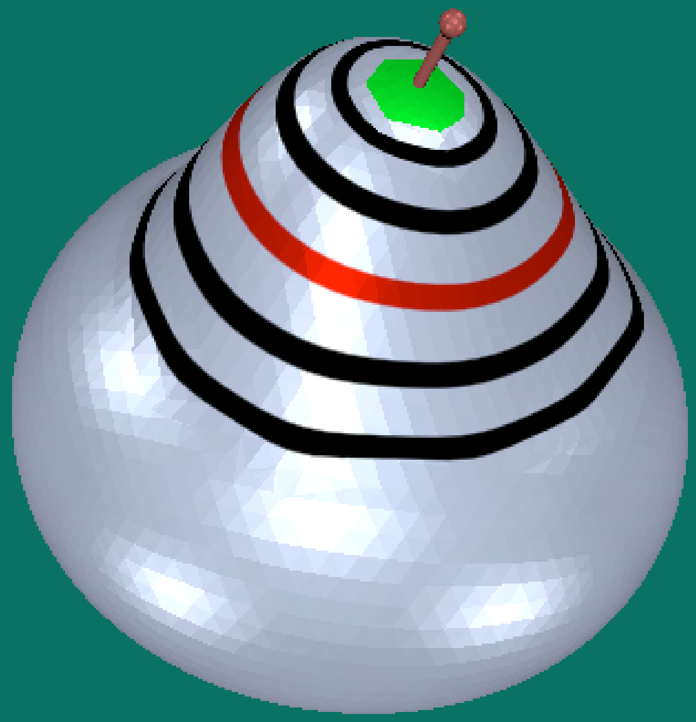




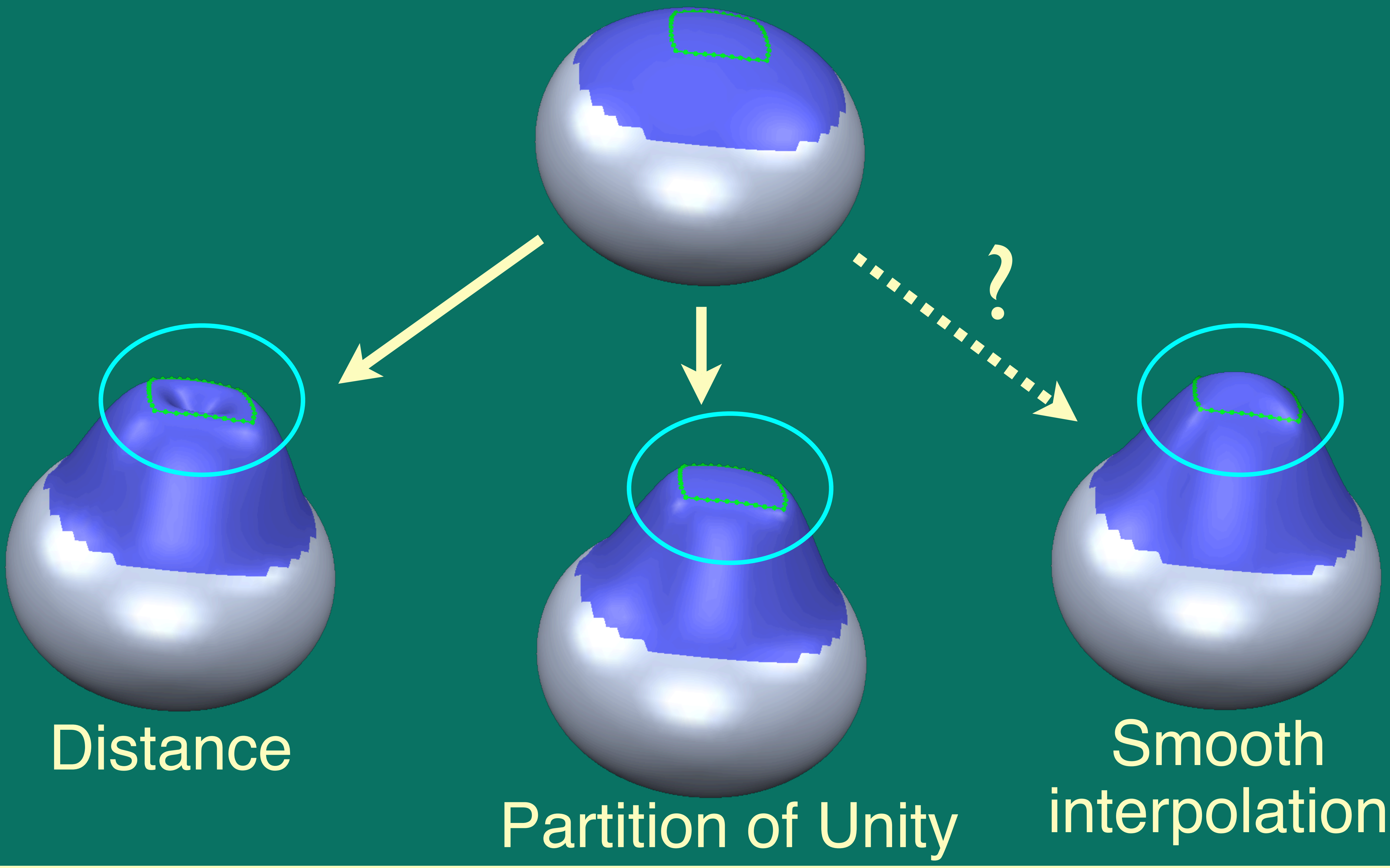
Distance-Based Propagation

- Propagate handle transformation
  - Euclidean / geodesic distance

- Surface properties?
- Interpolation constraints?



Distance-Based Propagation



Boundary Constraint Modeling

- Constrained energy minimization
  - Moreton & Sequin 1992
  - Welch & Witkin 1992
  - Kobbelt et al. 1998
- Boundary element method
  - James & Pai 1999
- Alternative detail representation
  - Sorkine et al. 2004
  - Yu et al. 2004

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Boundary Constraint Modeling

- Prescribe boundary constraints
  - Vertex positions
  - Vertex continuities

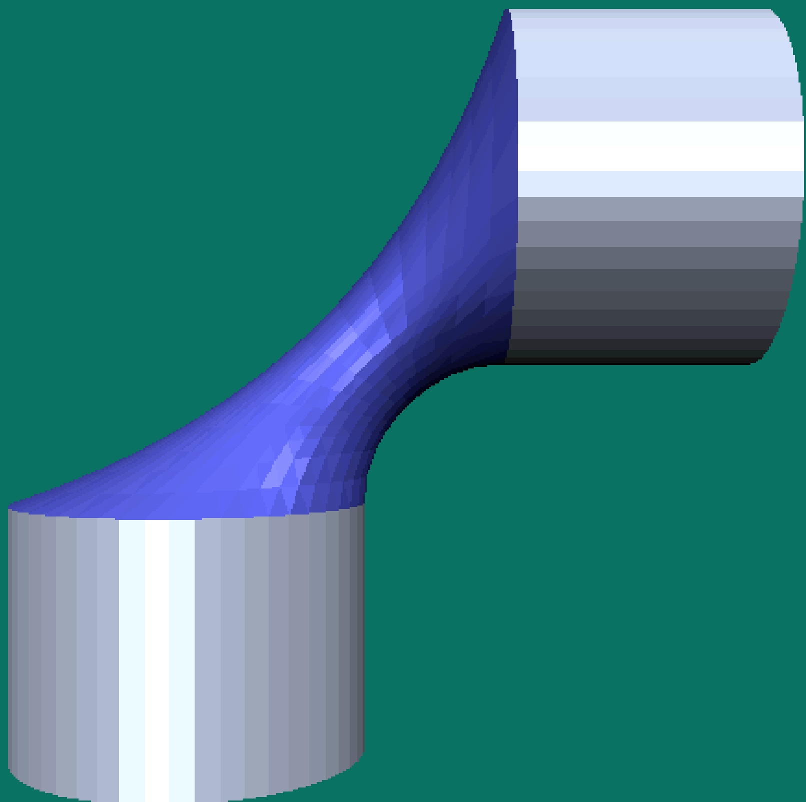
- Constraint energy minimization

$$E_k(S) = \int F_k\left(S_{u^k}, S_{u^{k-1}v}, \ldots, S_{v^k}\right)$$

- Euler-Lagrange PDE:

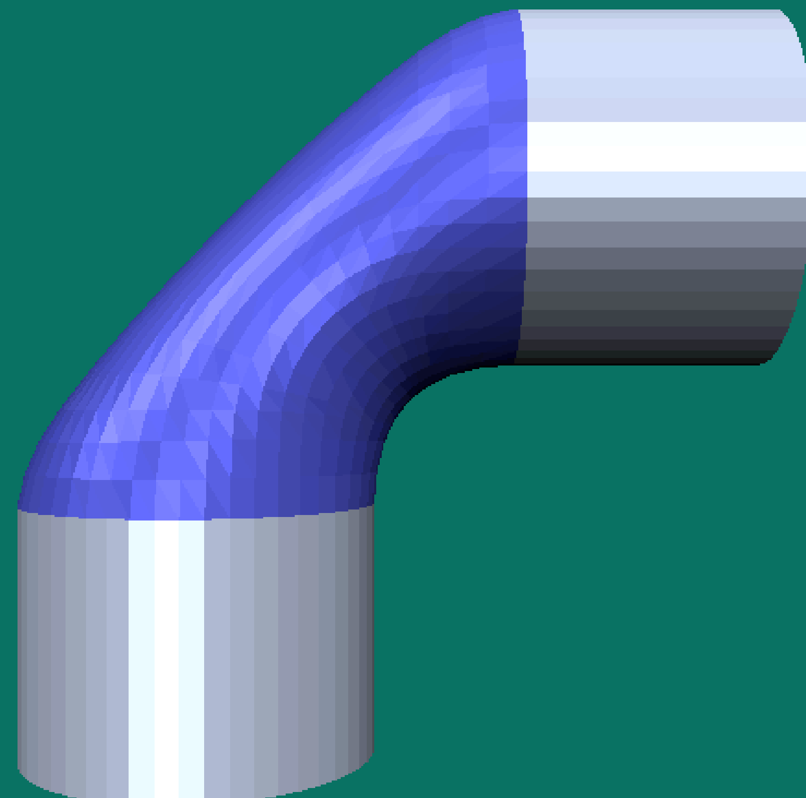
$$\Delta^k(S) = 0$$

Energy Functionals



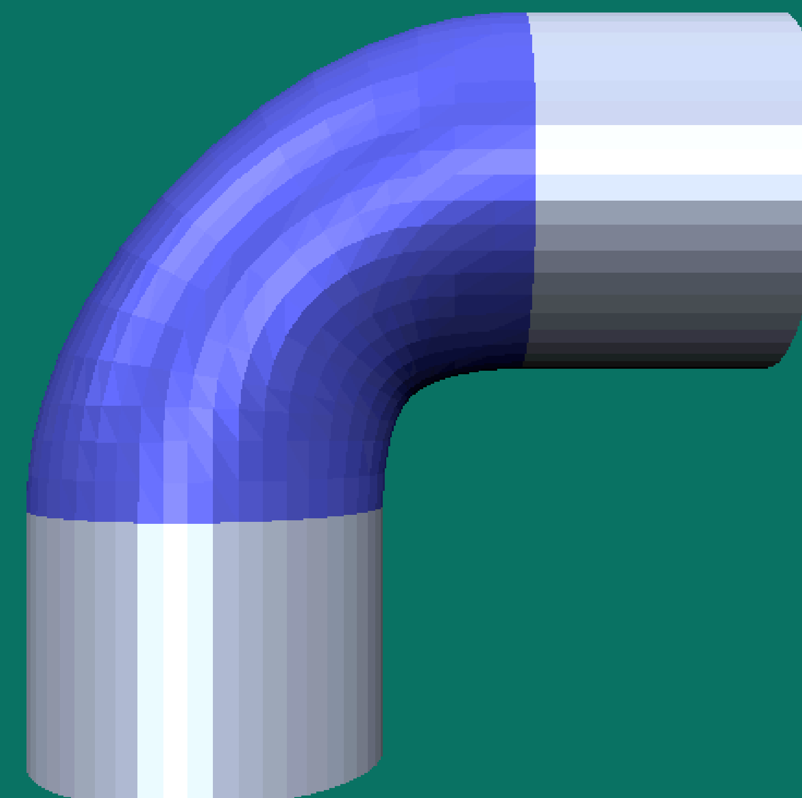
Membrane

$$\Delta S = 0$$



Thin-Plate

$$\Delta^2 S = 0$$

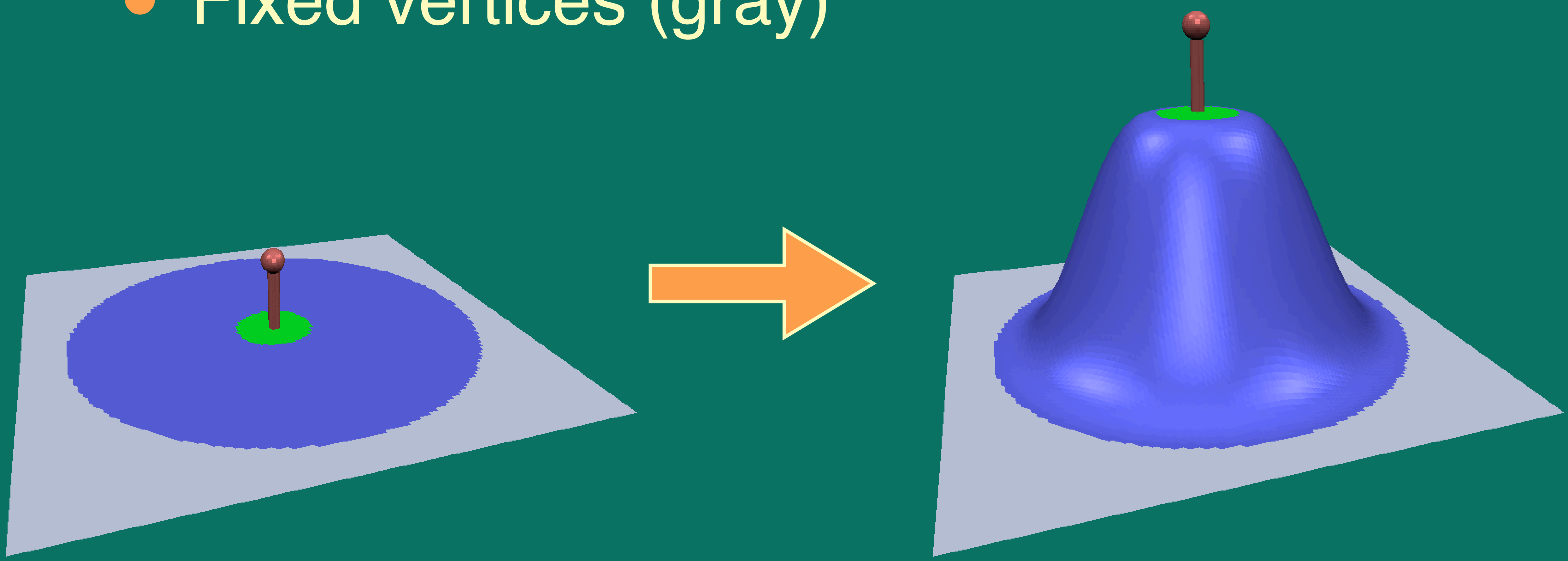


MVS

$$\Delta^3 S = 0$$

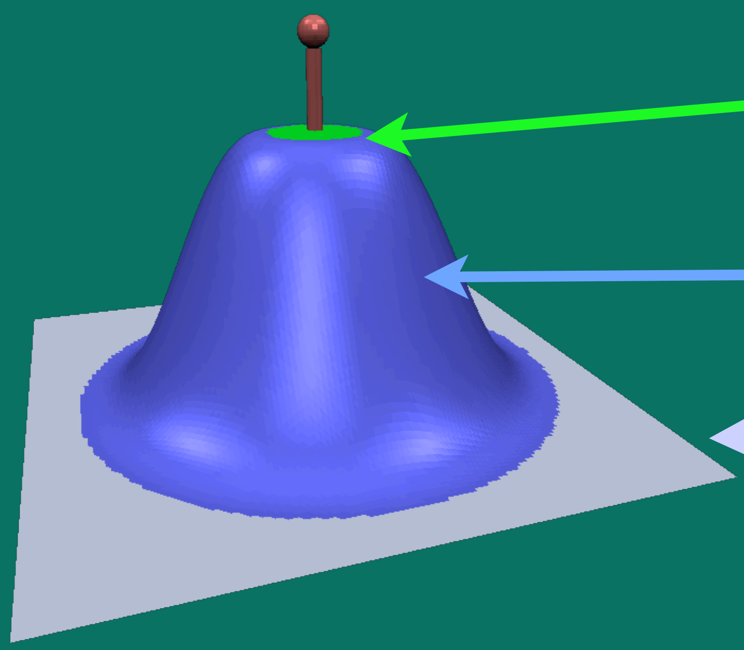
Modeling Metaphor [Kobbelt et al. 98]

- Support region (blue)
- Handle regions (green)
- Fixed vertices (gray)



Linear System

[Kobbelt et al. 98]



$\mathbf{h} = \{h_1, \ldots, h_H\}$ (k rings only)

$\mathbf{p} = \{p_1, \ldots, p_P\}$

$\mathbf{f} = \{f_1, \ldots, f_F\}$ (k rings only)

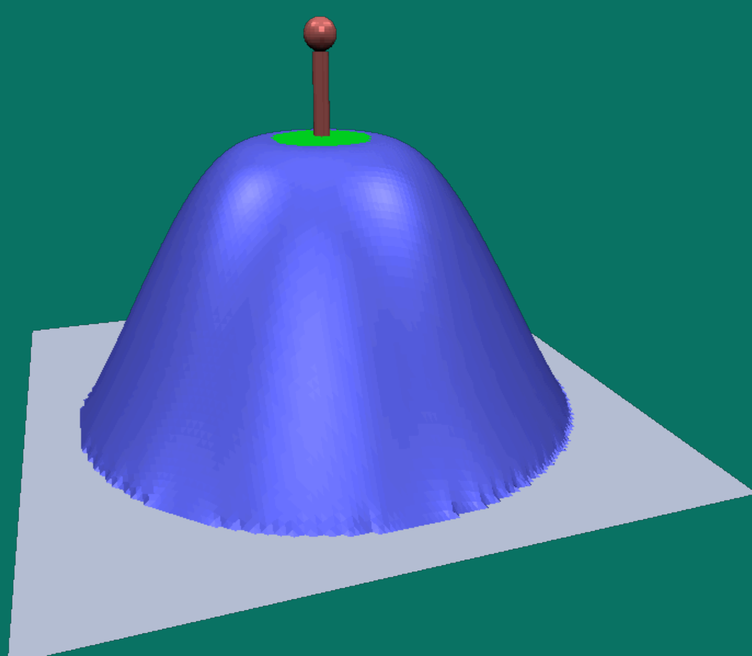
$$\left(\begin{array}{ccc} & \Delta^k & \\ \hline 0 & I_F & 0 \\ 0 & 0 & I_H \end{array} \right) \left(\begin{array}{c} \mathbf{p} \\ \mathbf{f} \\ \mathbf{h} \end{array} \right) = \left(\begin{array}{c} \mathbf{0} \\ \mathbf{f} \\ \mathbf{h} \end{array} \right)$$

Industrial Evaluation

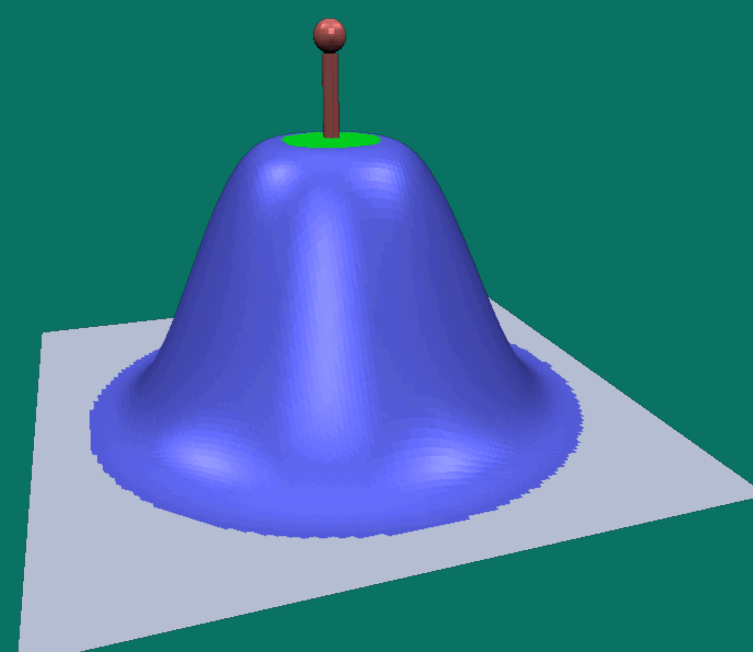
- [Kobbelt et al. 98] targeted primarily at conceptual design
- More control for engineering applications:
  - Specify boundary smoothness
  - Anisotropic bending behavior

Boundary Smoothness

How smooth does the deformed region blend with the fixed part?



C^0/C^2



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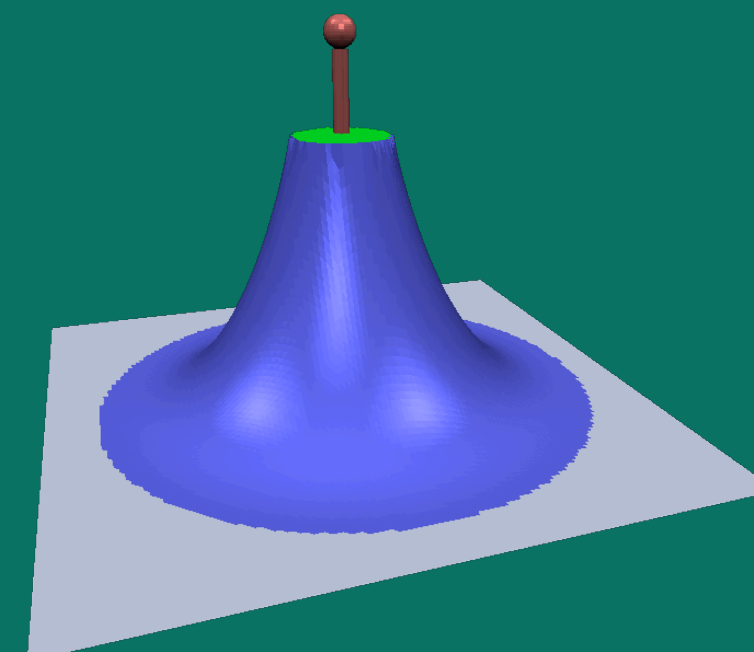


C^2/C^0

Boundary Smoothness

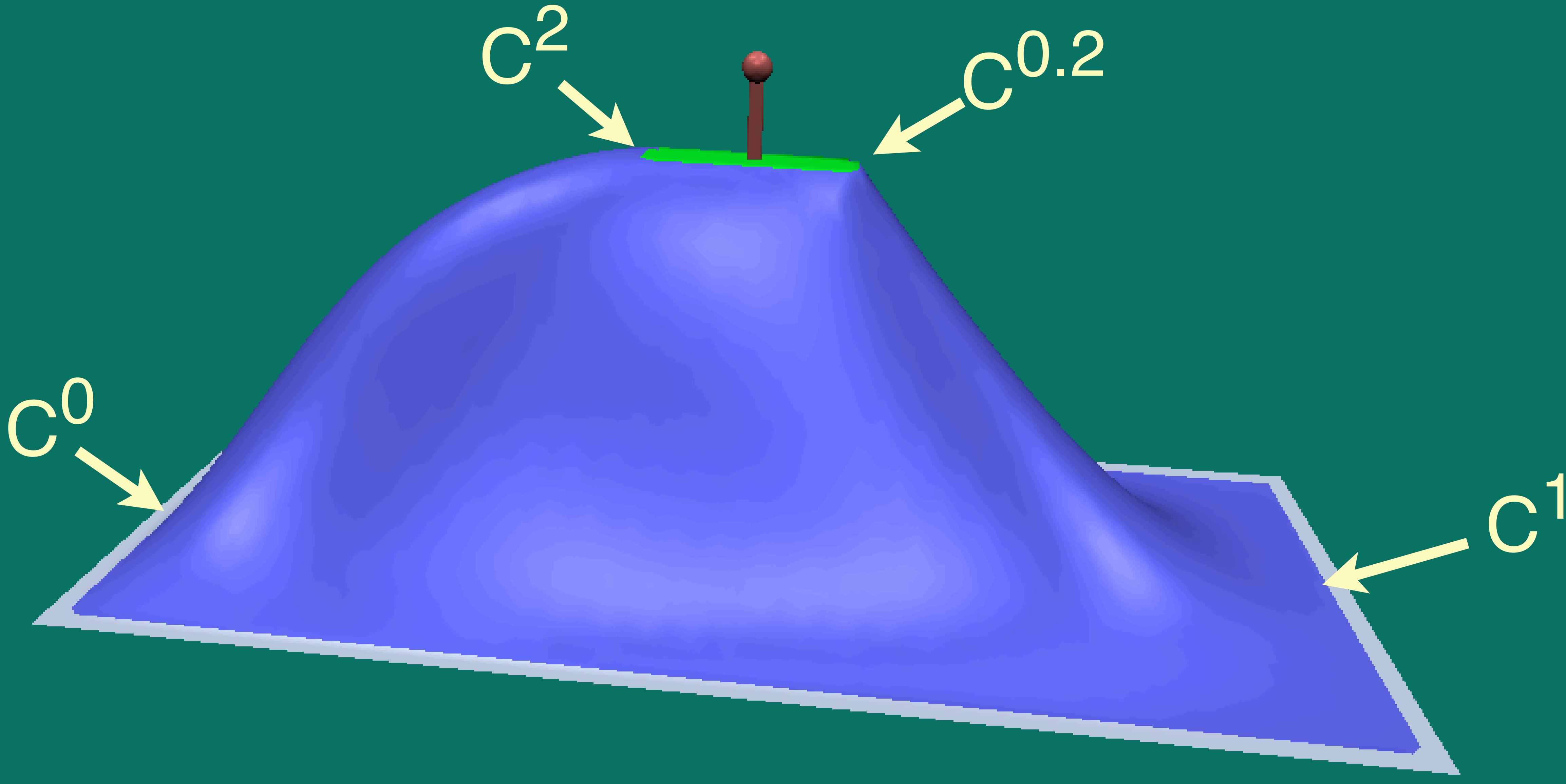
- Δ^k surfaces can do up to C^{k-1}
  - Real-valued smoothness $c(p) \in [0, k-1]$

- Adjust recursive Laplace definition

$$\bar{\Delta}^3(p) = \Delta\left(\lambda_2(p)\,\Delta\left(\lambda_1(p)\,\Delta(p)\right)\right)$$

$\lambda_2(p)$	$\lambda_1(p)$	
0	$[0,1]$	⇒ $C^{0+\lambda_1(p)}$
$[0,1]$	1	⇒ $C^{1+\lambda_2(p)}$

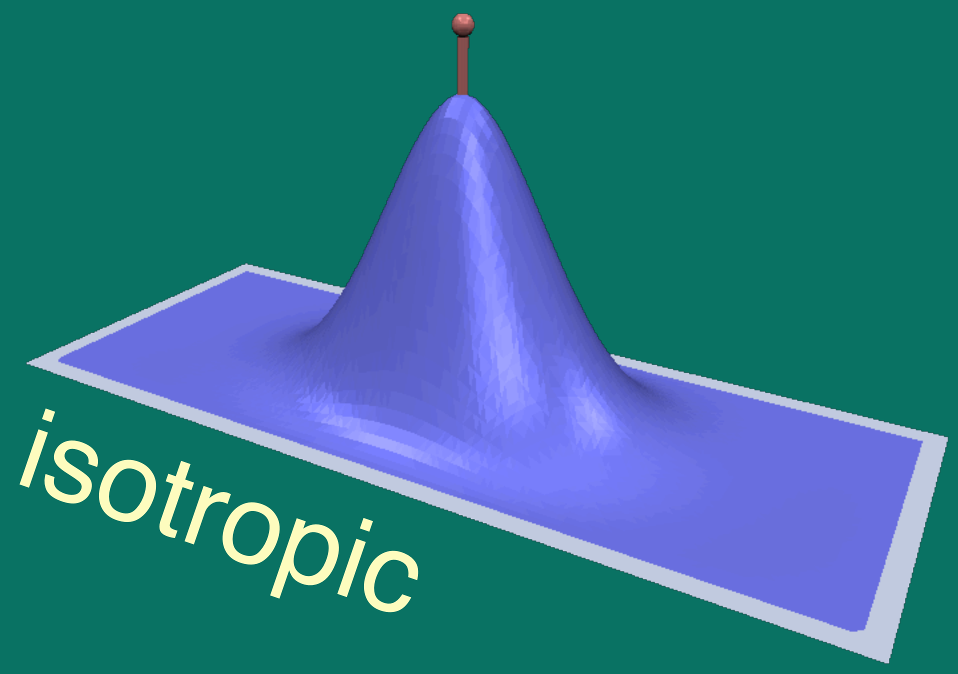
Boundary Smoothness

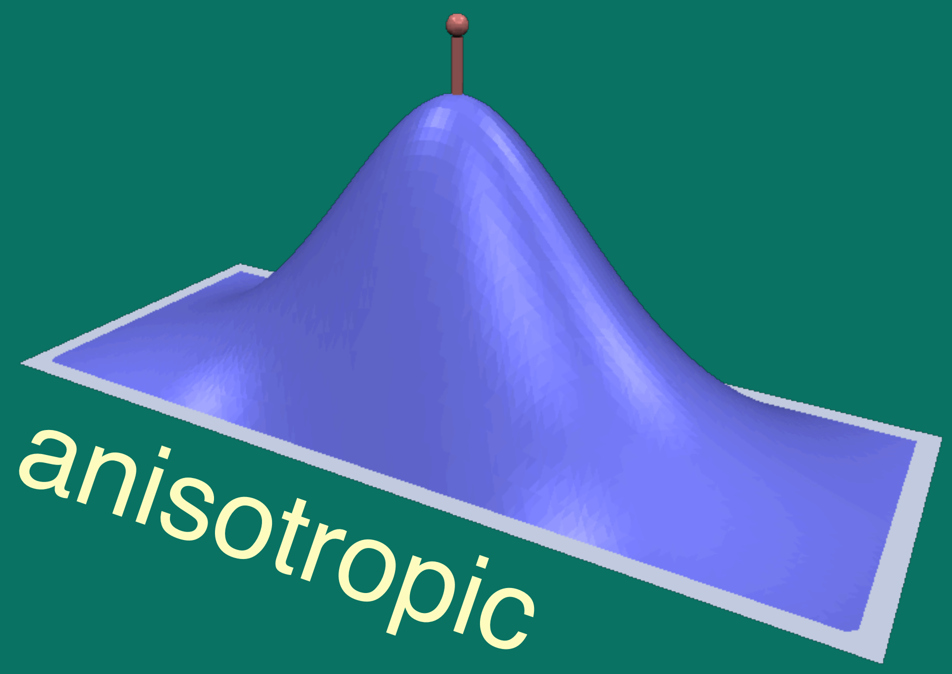


Segment-wise boundary smoothness

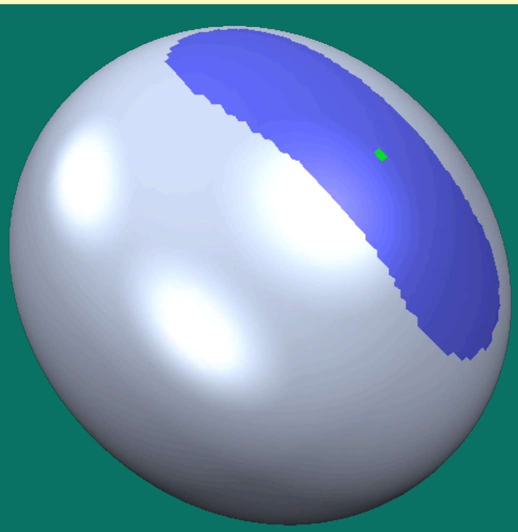
Anisotropic Bending

- Isotropic bending for standard Laplace
- Not intuitive for anisotropic support region
- Bending should adapt to support's shape



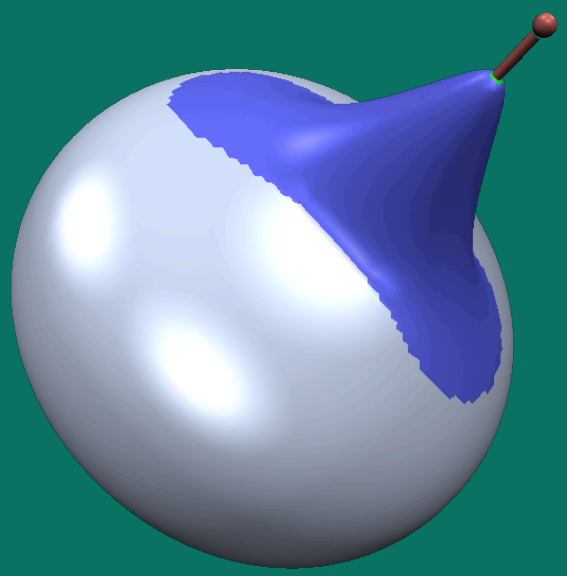


Anisotropic Bending

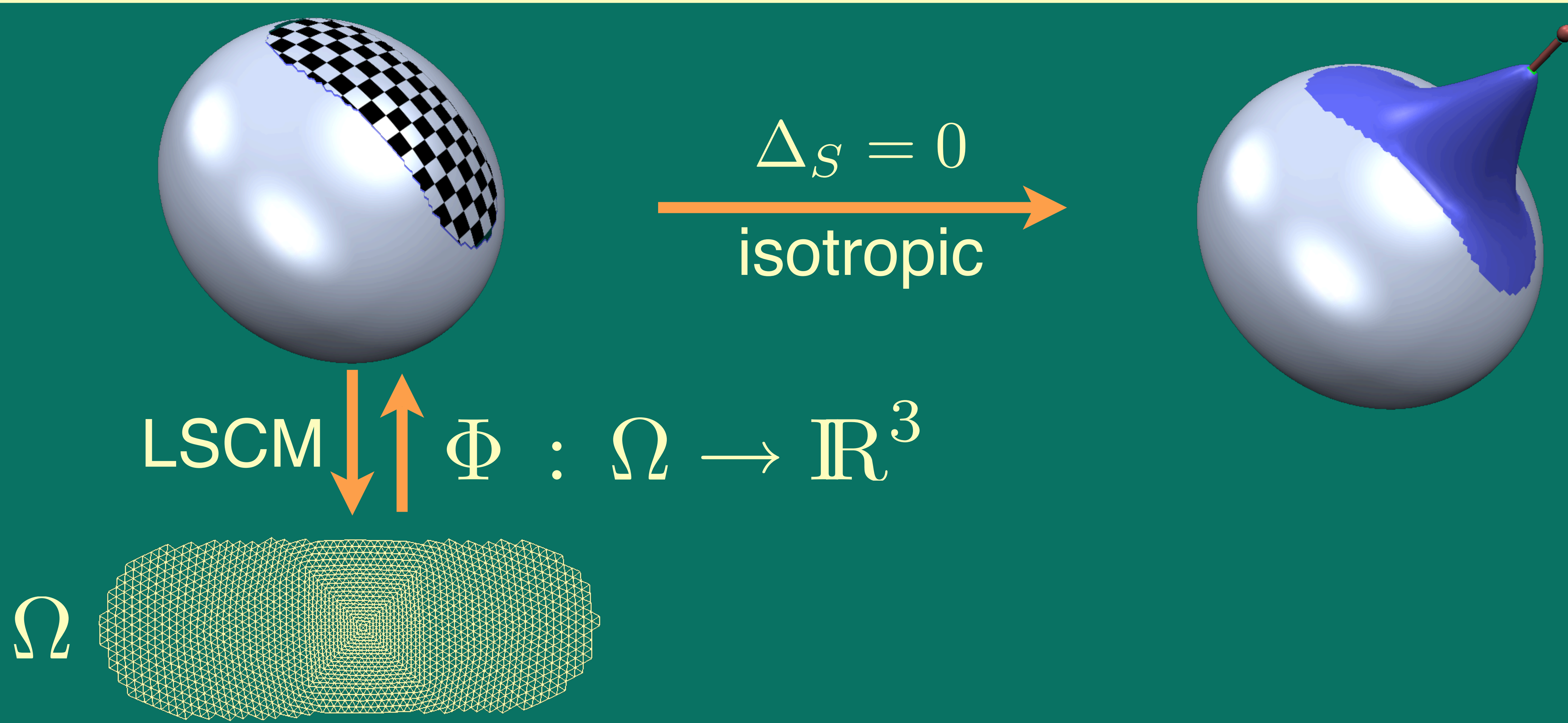


$$\Delta_S = 0$$

isotropic



Anisotropic Bending



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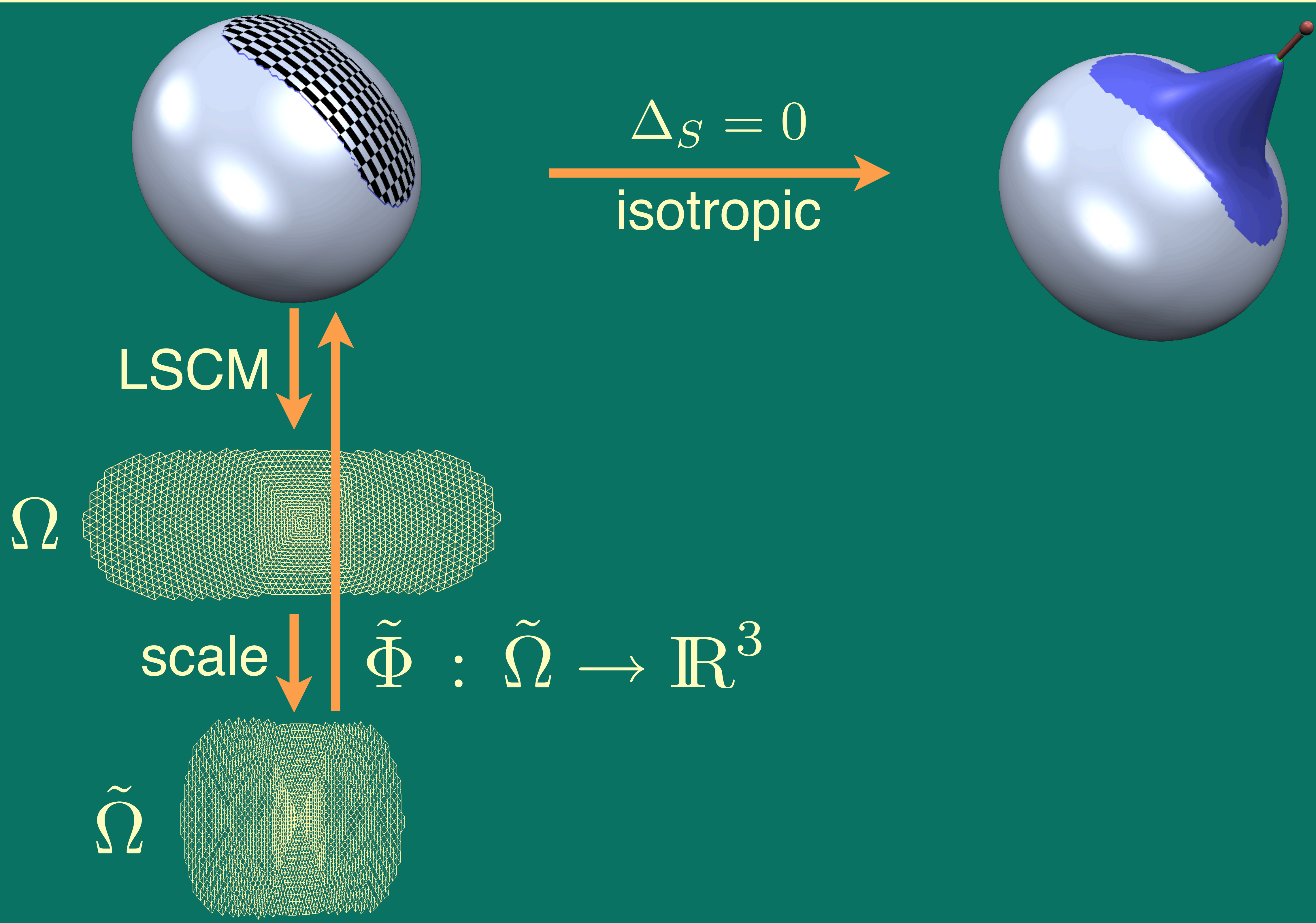

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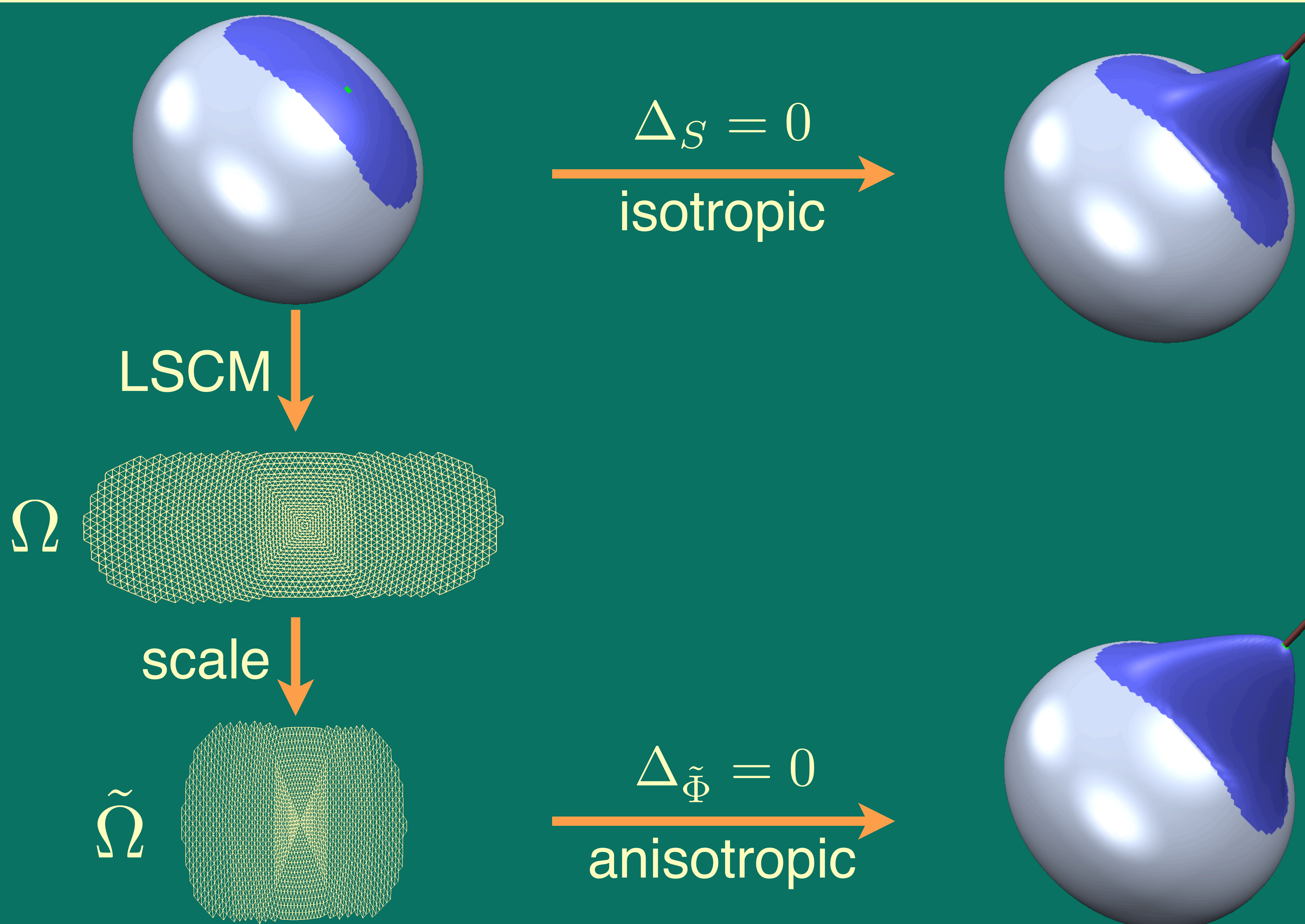


Real-Time Modeling

$$\begin{pmatrix} & \bar{\Delta}^k & \\ \hline 0 & I_F & 0 \\ 0 & 0 & I_H \end{pmatrix} \begin{pmatrix} \mathbf{p} \\ \mathbf{f} \\ \mathbf{h} \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \mathbf{f} \\ \mathbf{h} \end{pmatrix}$$

- Solve linear system each frame:
  - Too slow even for multi-grid solvers (#p>20k)
- System changes if certain constraints change
- Precompute per-handle basis function

Precomputed Basis Functions

- System to be solved

$$\underbrace{\begin{pmatrix} & \bar{\Delta}^k & \\ \hline 0 & I_F & 0 \\ 0 & 0 & I_H \end{pmatrix}}_{=:L} \begin{pmatrix} \mathbf{p} \\ \mathbf{f} \\ \mathbf{h} \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \mathbf{f} \\ \mathbf{h} \end{pmatrix}$$

- Columns of inverse are bases

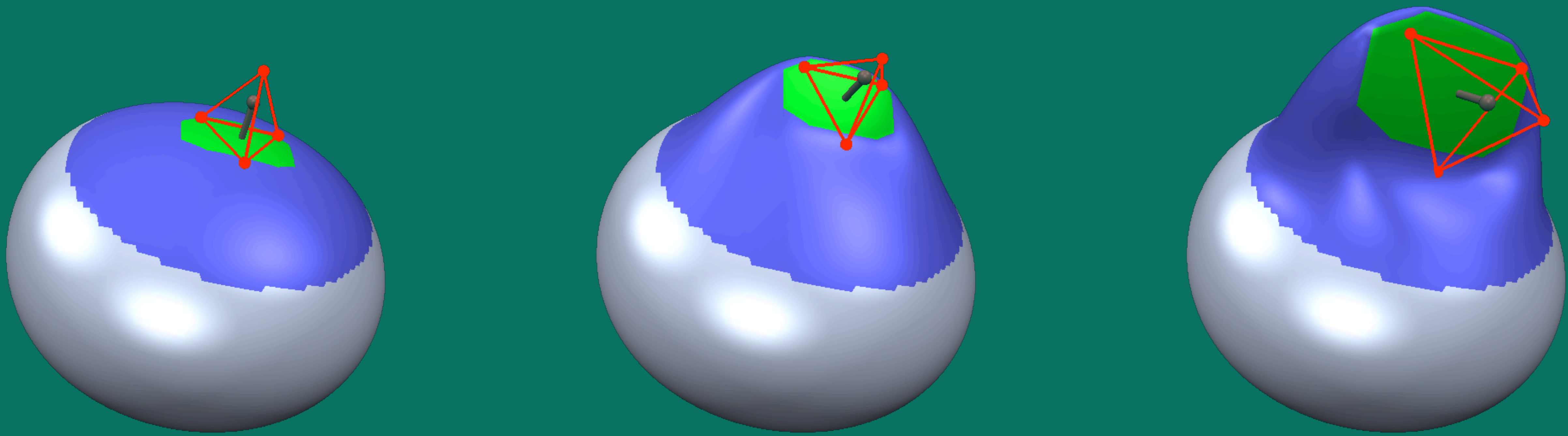
$$\begin{pmatrix} \mathbf{p} \\ \mathbf{f} \\ \mathbf{h} \end{pmatrix} = L^{-1} \begin{pmatrix} \mathbf{0} \\ \mathbf{f} \\ \mathbf{0} \end{pmatrix} + L^{-1} \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{h} \end{pmatrix}$$

Precomputed Basis Functions

- Simple user interaction
  - Handle is transformed affinely only

- Represent handle points w.r.t. affine frame

$$\mathbf{h} = \mathbf{Q}\ [a, b, c, d]^T$$



Precomputed Basis Functions

$$\begin{pmatrix} \mathbf{p} \\ \mathbf{f} \\ \mathbf{h} \end{pmatrix} = L^{-1} \begin{pmatrix} \mathbf{0} \\ \mathbf{f} \\ \mathbf{0} \end{pmatrix} + L^{-1} \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \\ \boxed{\mathbf{h}} \end{pmatrix}$$

Precomputed Basis Functions

$$\begin{pmatrix} \mathbf{p} \\ \mathbf{f} \\ \mathbf{h} \end{pmatrix} = L^{-1} \begin{pmatrix} \mathbf{0} \\ \mathbf{f} \\ \mathbf{0} \end{pmatrix} + L^{-1} \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{Q} \end{pmatrix} [a, b, c, d]^T .$$

Precomputed Basis Functions

$$\begin{pmatrix} \mathbf{p} \\ \mathbf{f} \\ \mathbf{h} \end{pmatrix} = \mathbf{c} + \mathbf{B}\,[a, b, c, d]^T.$$

- Solve linear system
  - 3 times (const. part) → $\mathbf{c} \in \mathbb{R}^{N \times 3}$
  - 4 times per handle → $\mathbf{B} \in \mathbb{R}^{N \times 4}$
- Moderate precomputation times
  - 10k: 1 sec, 50k: 9s, O(#p)

Precomputed Basis Functions

- Real-time per-frame solution

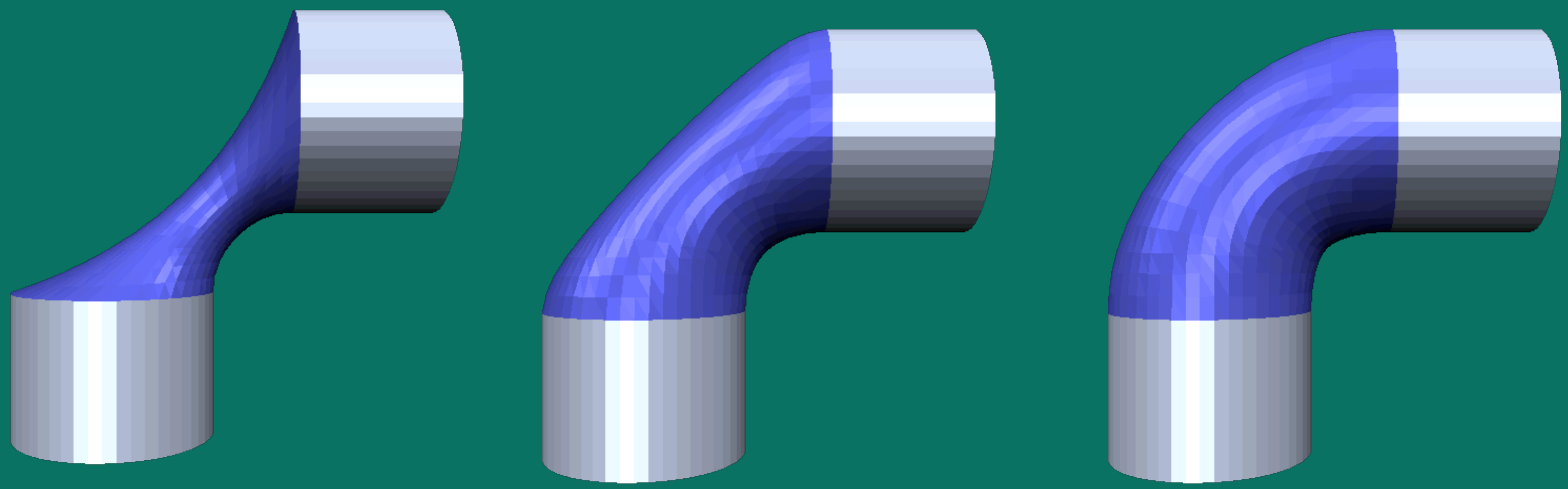
$$\mathbf{p}' = \mathbf{c} + \mathbf{B}\,[a', b', c', d']^T$$

- Custom-tailored basis function

$$S' = S + \mathbf{B}\,\underbrace{[\delta a, \delta b, \delta c, \delta d]}_{\delta C}^T$$

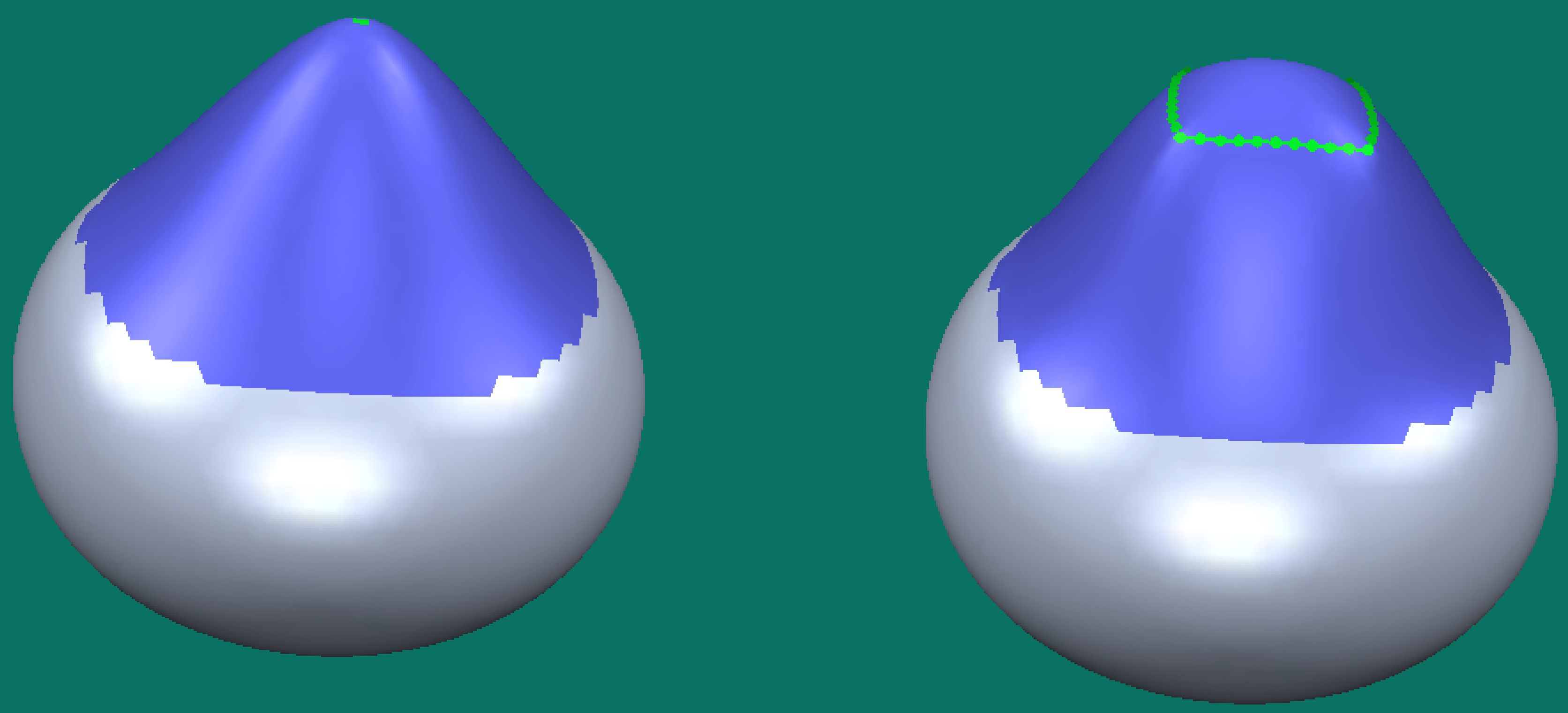
Custom-Tailored Basis Function

- Energy functional (*stiffness*)



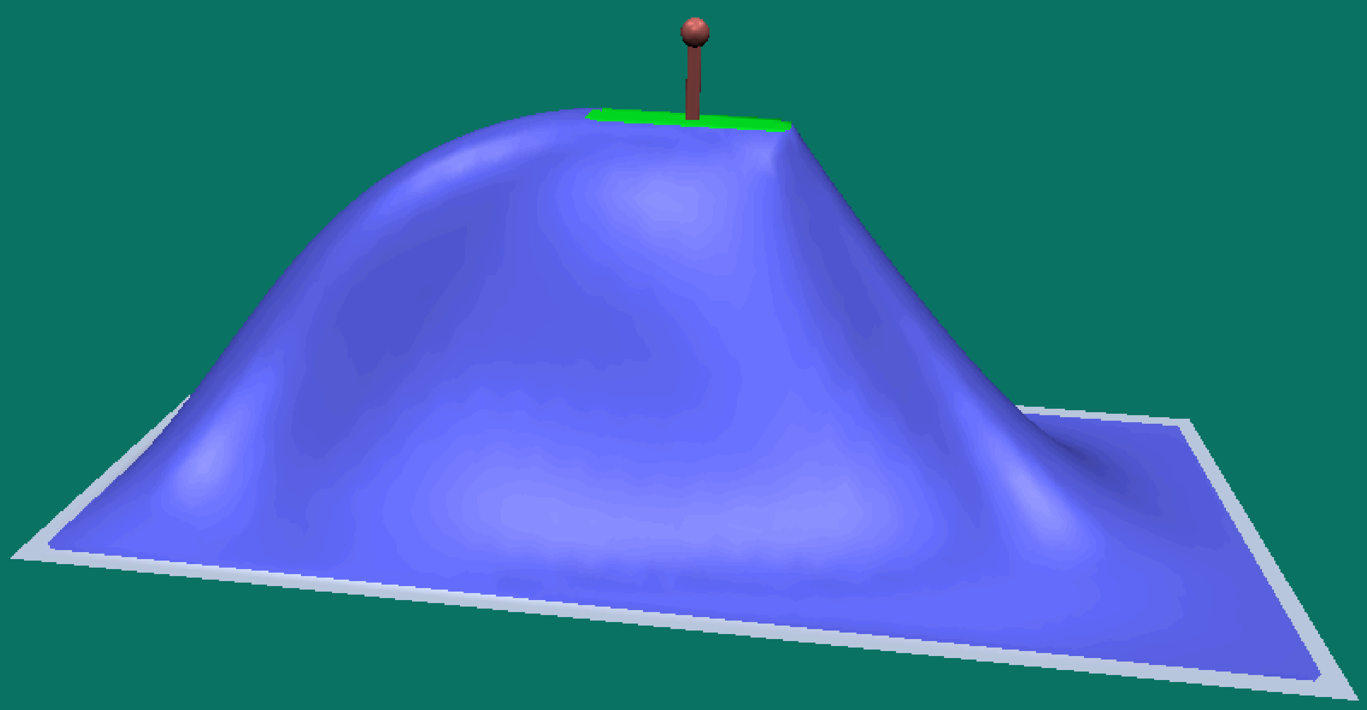
Custom-Tailored Basis Function

- Energy functional (*stiffness*)
- Support & handle (*fullness*)



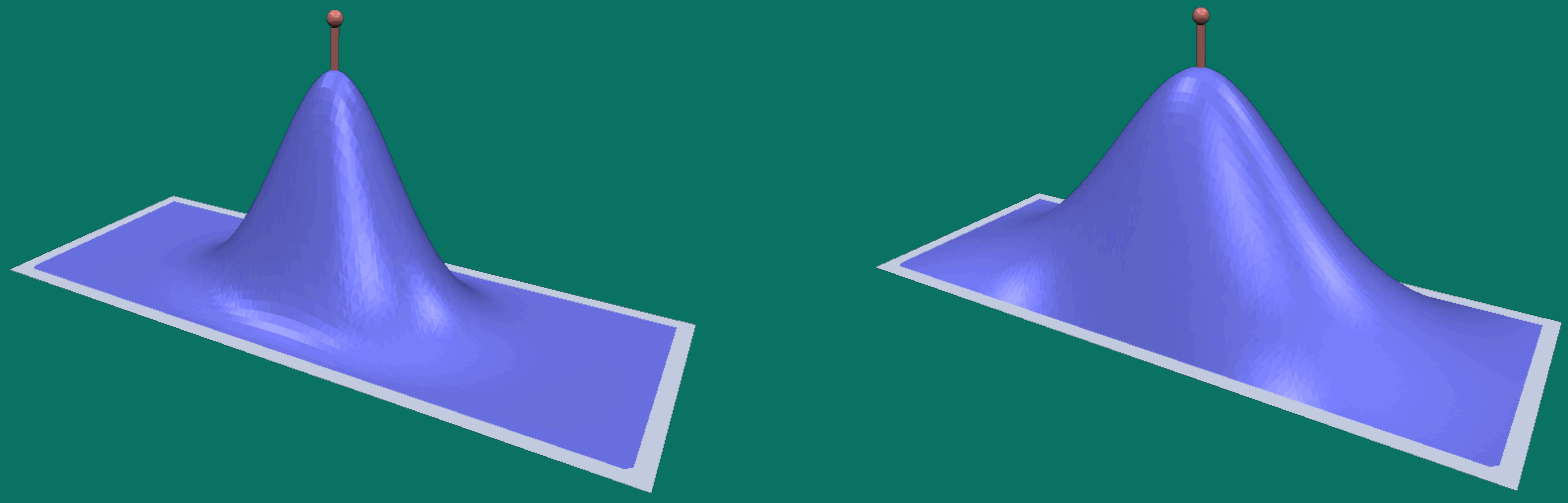
Custom-Tailored Basis Function

- Energy functional (*stiffness*)
- Support & handle (*fullness*)
- Boundary smoothness



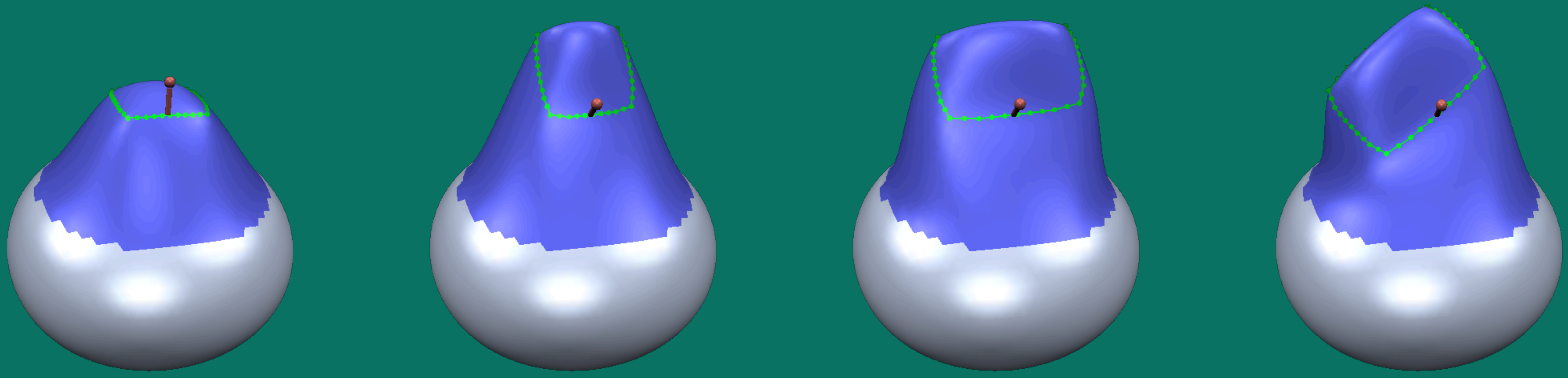
Custom-Tailored Basis Function

- Energy functional (*stiffness*)
- Support & handle (*fullness*)
- Boundary smoothness
- Isotropic / anisotropic



Custom-Tailored Basis Function

- Energy functional (*stiffness*)
- Support & handle (*fullness*)
- Boundary smoothness
- Isotropic / anisotropic
- Arbitrary affine handle transformation



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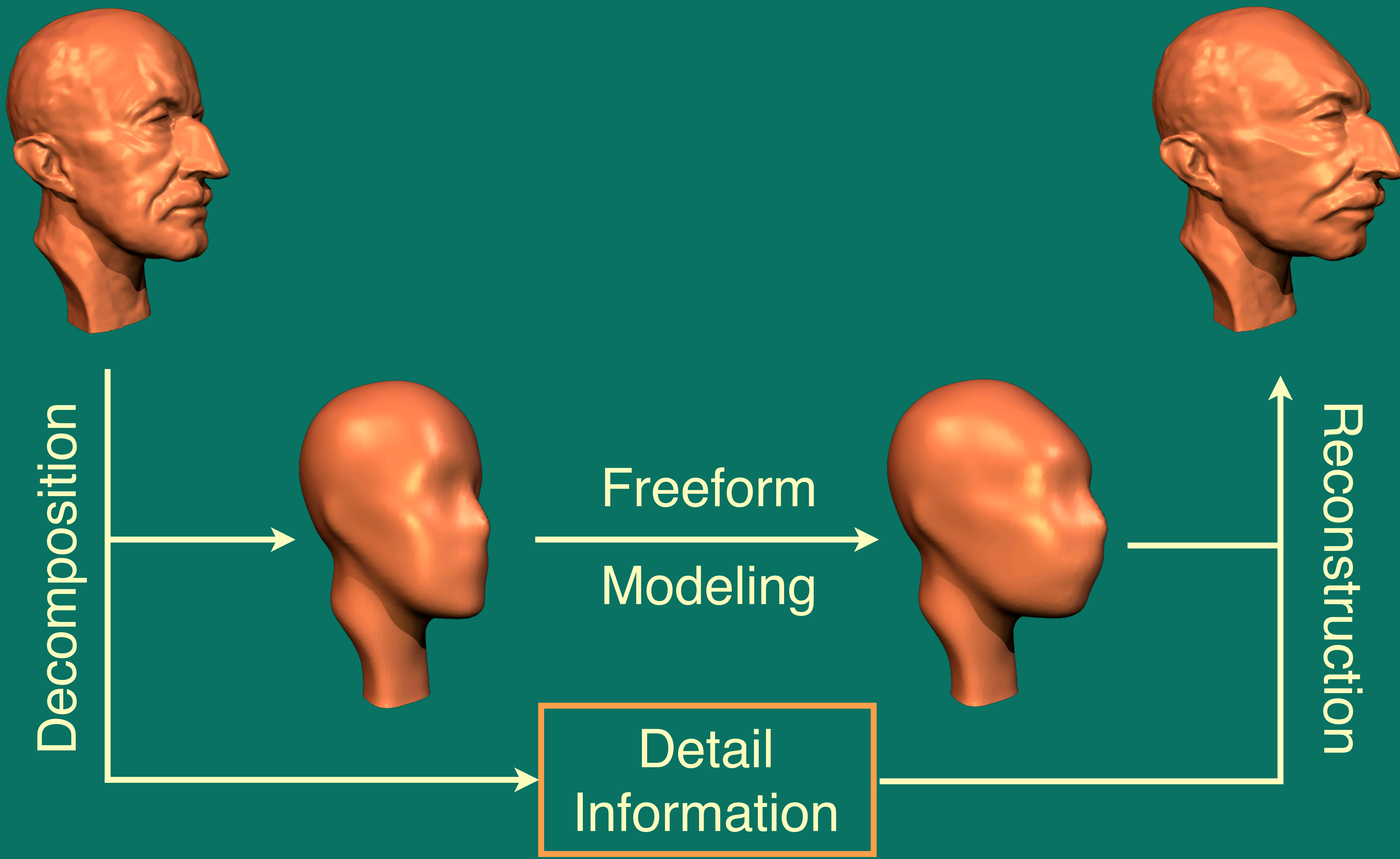
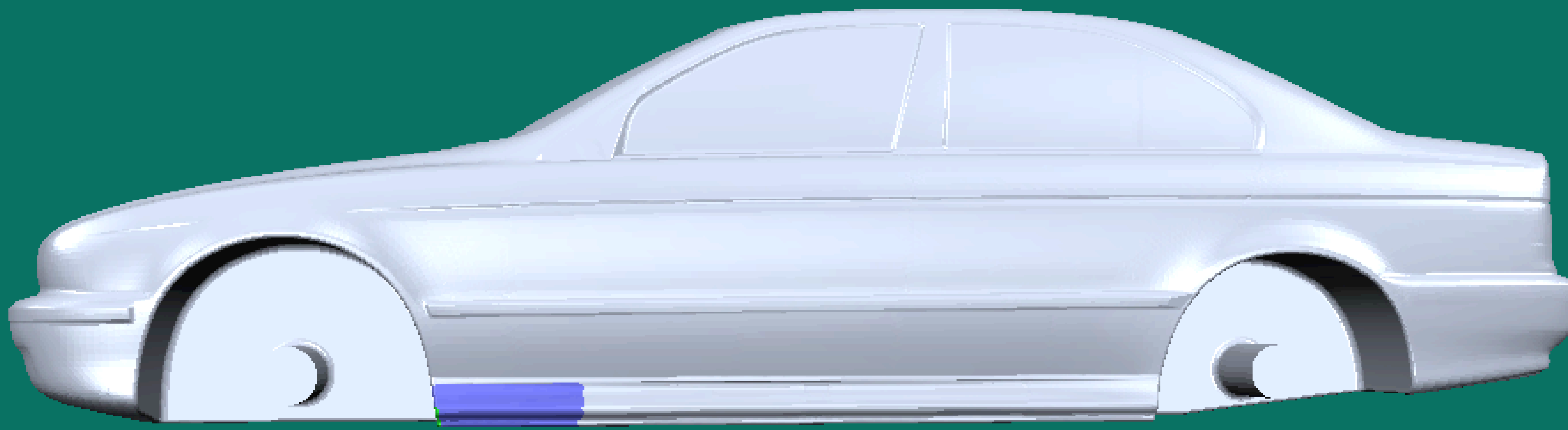
Multiresolution Modeling

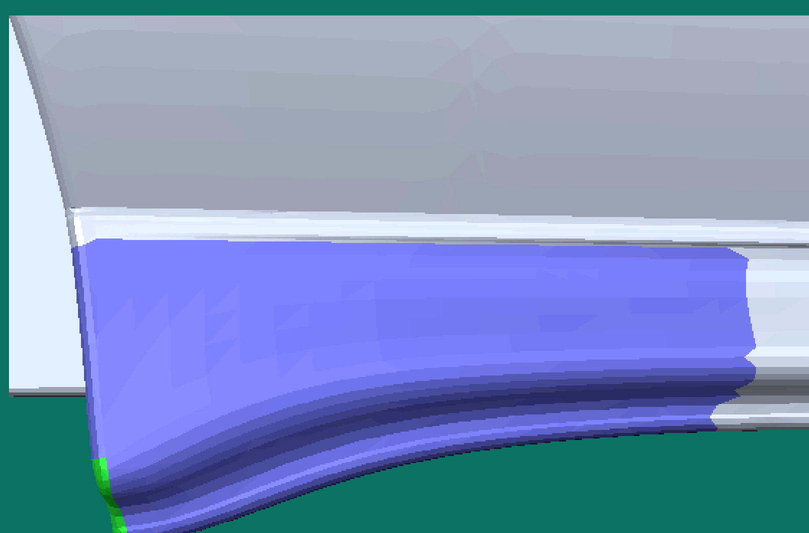
- Freeform modeling builds <u>smooth</u> surfaces
- Real-world models have fine details
  - Modify existing models
- Integrate into multiresolution framework

Multiresolution Modeling

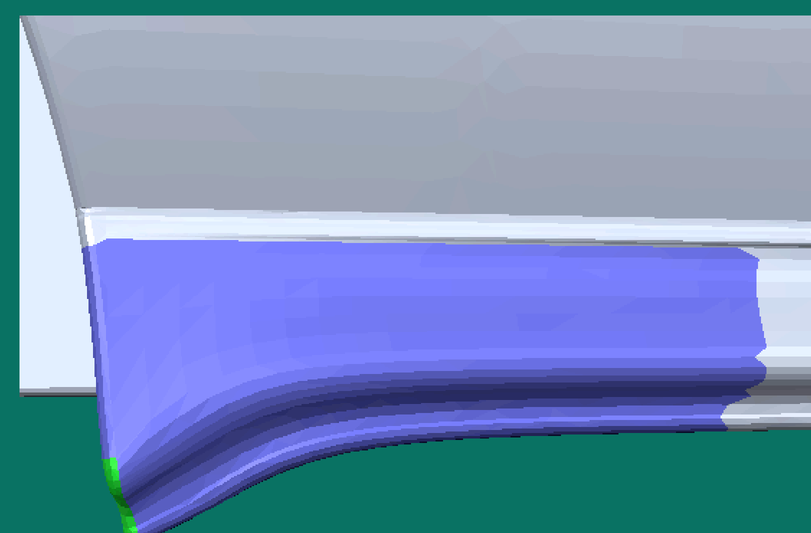


Sillboard

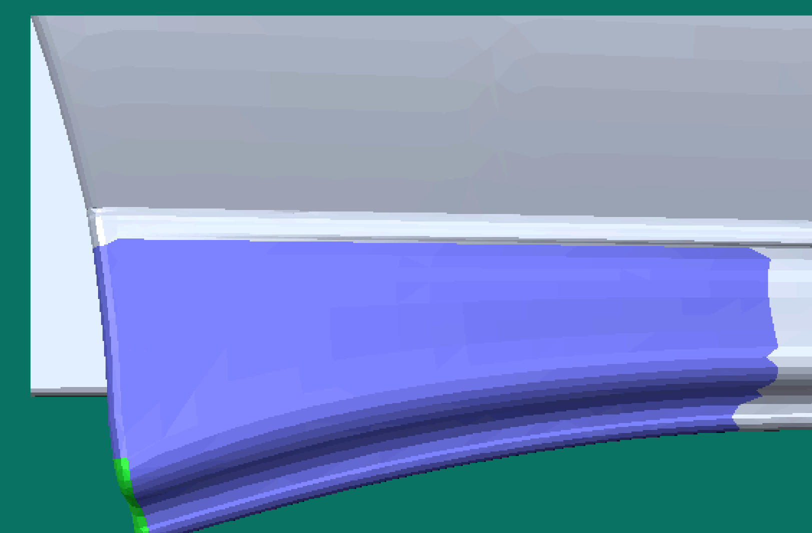




C^1/C^1
isotropic

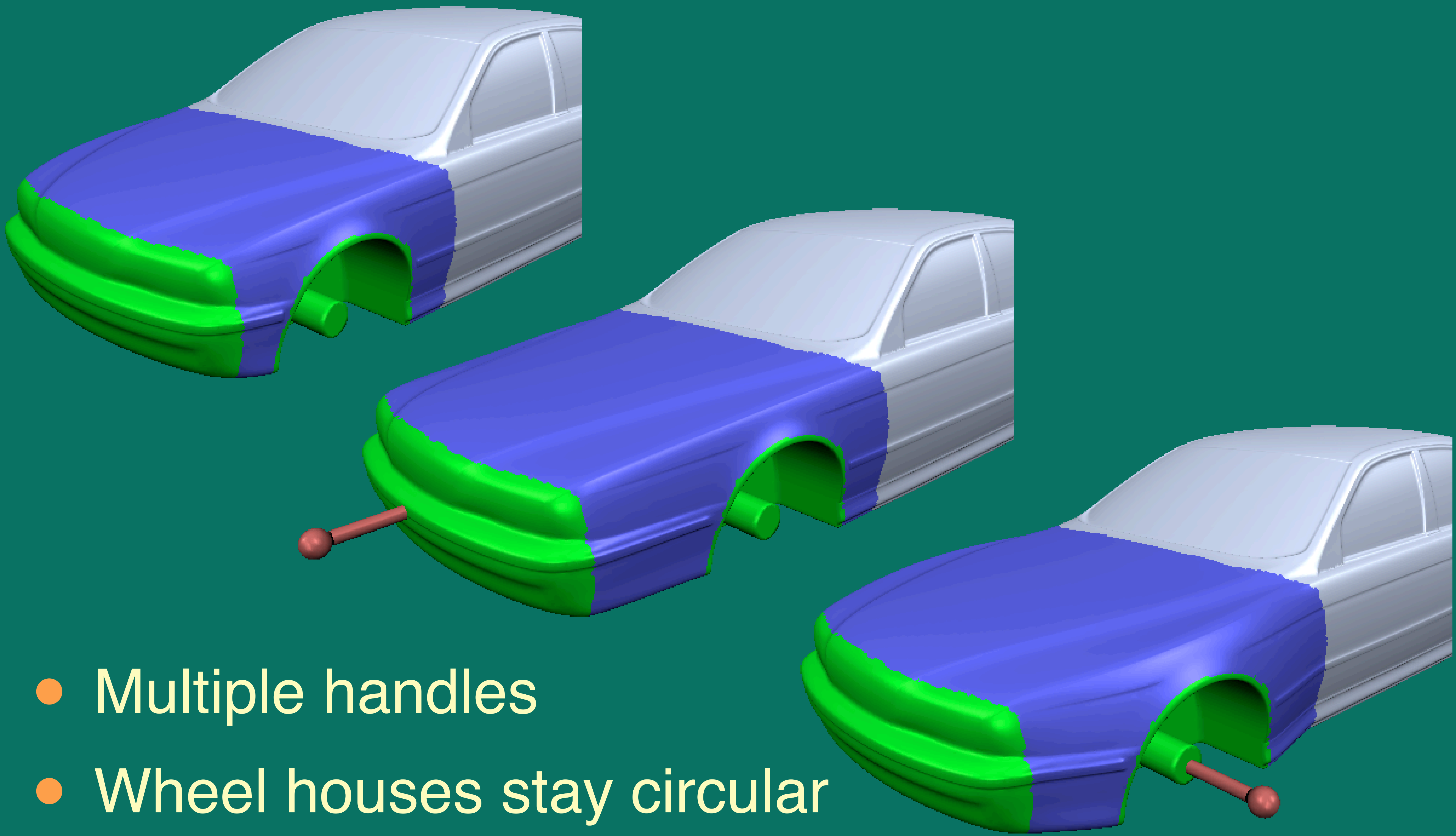


C^1/C^0
isotropic



C^1/C^0
anisotropic

Stretching the Hood



- Multiple handles
- Wheel houses stay circular

Conclusion

- Custom-tailored basis functions
  - Simple user interaction
- Boundary constraint modeling
  - Boundary smoothness
  - Anisotropic bending
- Precomputed basis functions
  - Real-time deformations

Future Work

- Topological flexibility
  - Anisotropy requires parameterization
  - Disk shaped support
- Numerical improvements
  - Robustness & efficiency
  - Remeshing approach, SGP 2004
- Dynamic remeshing